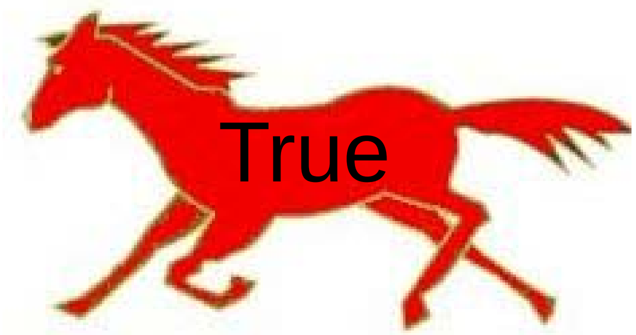
2011.08.29@ **消費者行動のダイナミクス研究部会**

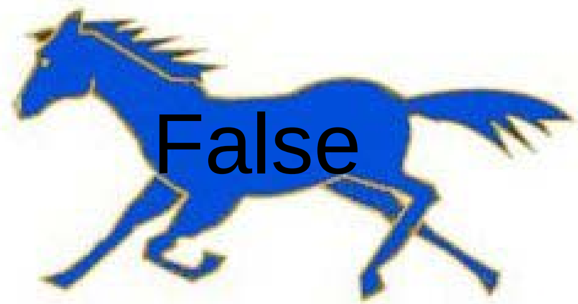
ヒトはいかに群れ、そして間違うのか

ヒトの意思決定の物理

守 真太郎 北里大学・理学部・物理学科
久門 正人 Standard and Poors

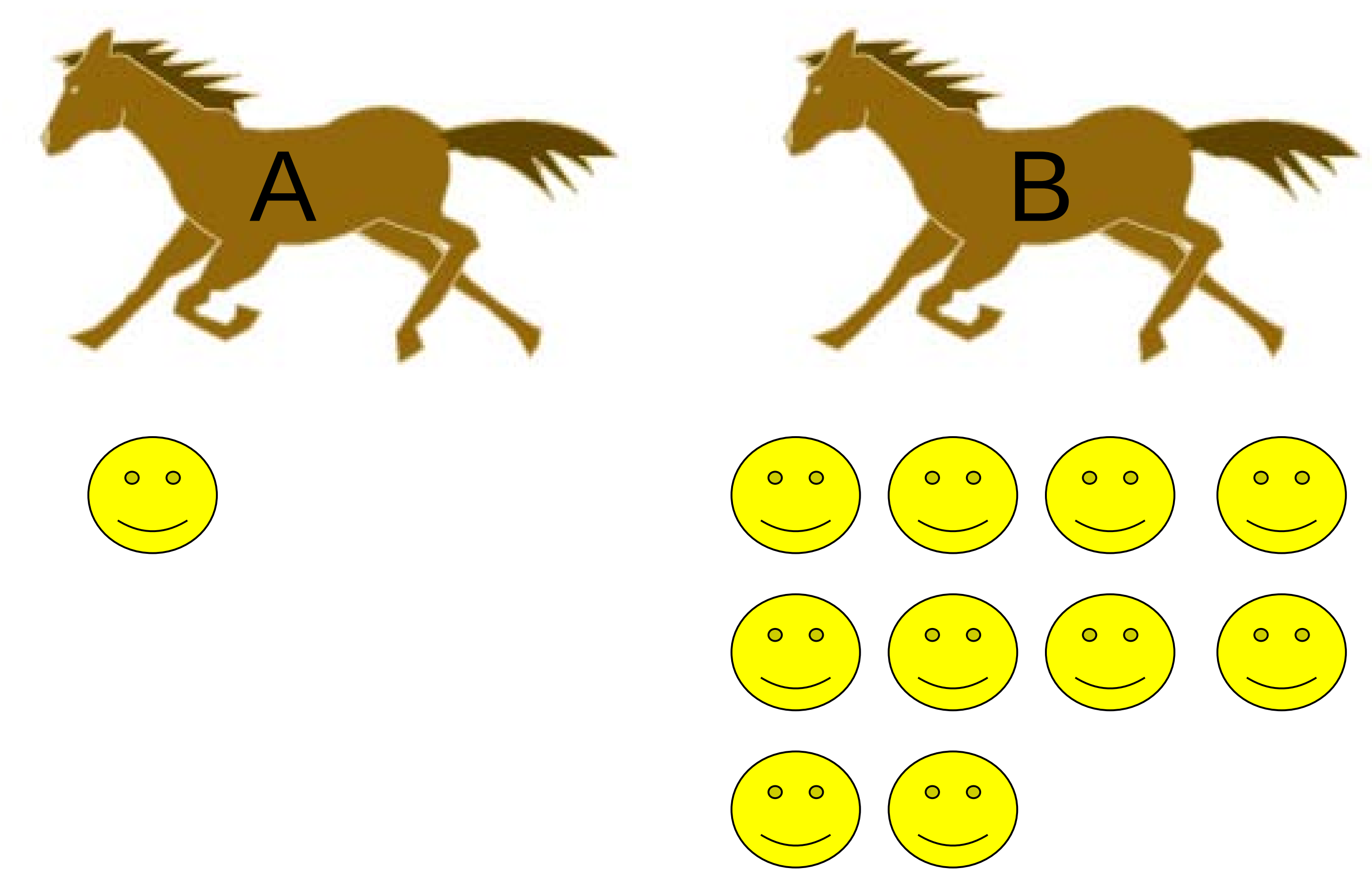
Quiz : A or B





Private Information

Public Knowledge



Can you trust “Public Knowledge” ?

Microscopic

How one decides ?

Phase Transition

Probability that “the Majority” is False.

α

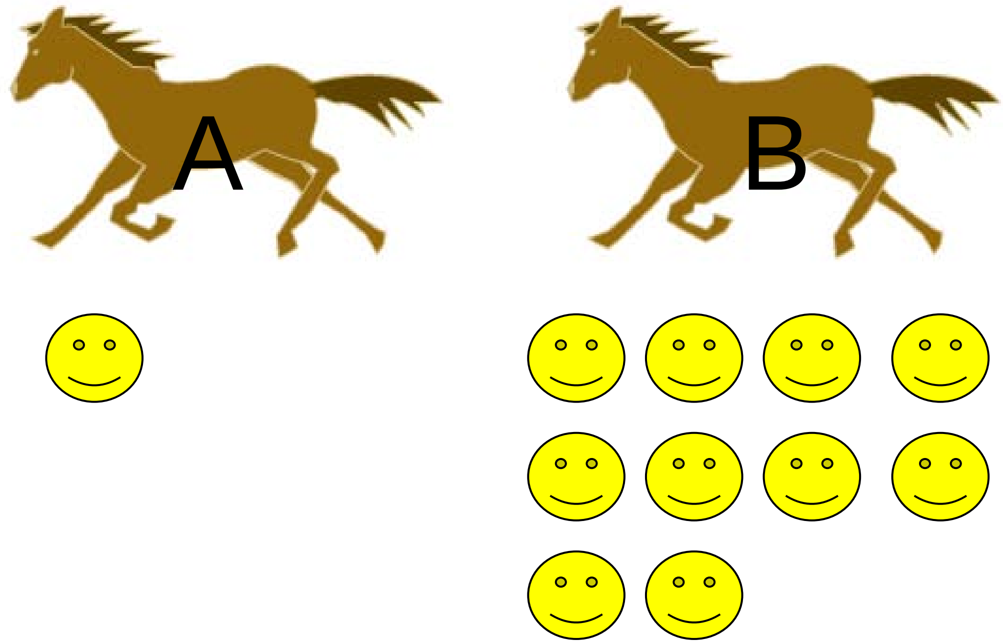
Non-analytic

Macroscopic

Information Cascade

- A.V. Banerjee.1992. S. Bikhchandani, D. Hirshleifer, I. Welch.(1992) I.Welch. June 1992.

Information Cascade



Private info. is well aggregated in the Public Knowledge ?

If Independent → Yes !

If Correlated → No !

(1) Sequential , One by One
(2) One can refer others' chices (not private info.)

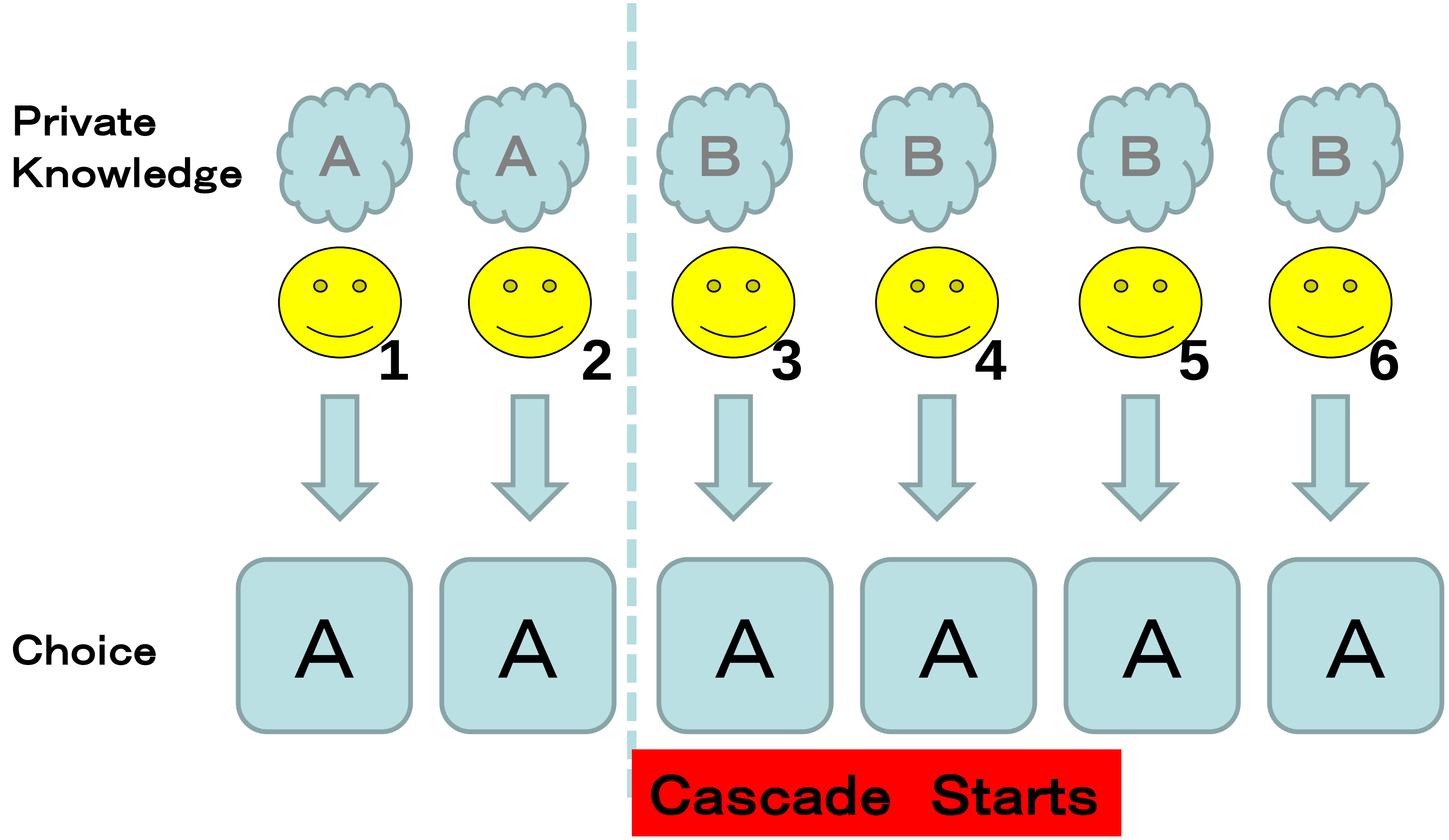


Table 2. Data for Selected Periods of Session 2

Period	Urn Used	Subject Number: Urn Decision (private draw) 1st round	2nd round	3rd round	4th round	5th round	6th round	Cascade Outcome
5	B	S12: A (*a*)	S11: B (*b*)	S9: B (*b*)	S7: B (*b*)	S8: B (*a*)	S10: B (*a*)	cascade
6	A	S12: A (*a*)	S8: A (*a*)	S9: A (*b*)	S11: A (*b*)	S10: A (*a*)	S7: A (*a*)	cascade
7	B	S8: B (*b*)	S7: A (*a*)	S10: B (*b*)	S11: B (*b*)	S12: B (*b*)	S9: B (*a*)	cascade
8	A	S8: A (*a*)	S9: A (*a*)	S12: B* (*b*)	S10: A (*a*)	S11: A (*b*)	S7: A (*a*)	cascade
9	B	S11: A (*a*)	S12: A (*a*)	S8: A (*b*)	S9: A (*b*)	S7: A (*b*)	S10: A (*b*)	reverse cascade

Key: Shading -- Bayesian decision, inconsistent with private information.

* -- Decision based on private information, inconsistent with Bayesian updating.

L. R. Anderson, C. A. Holt. 1996.

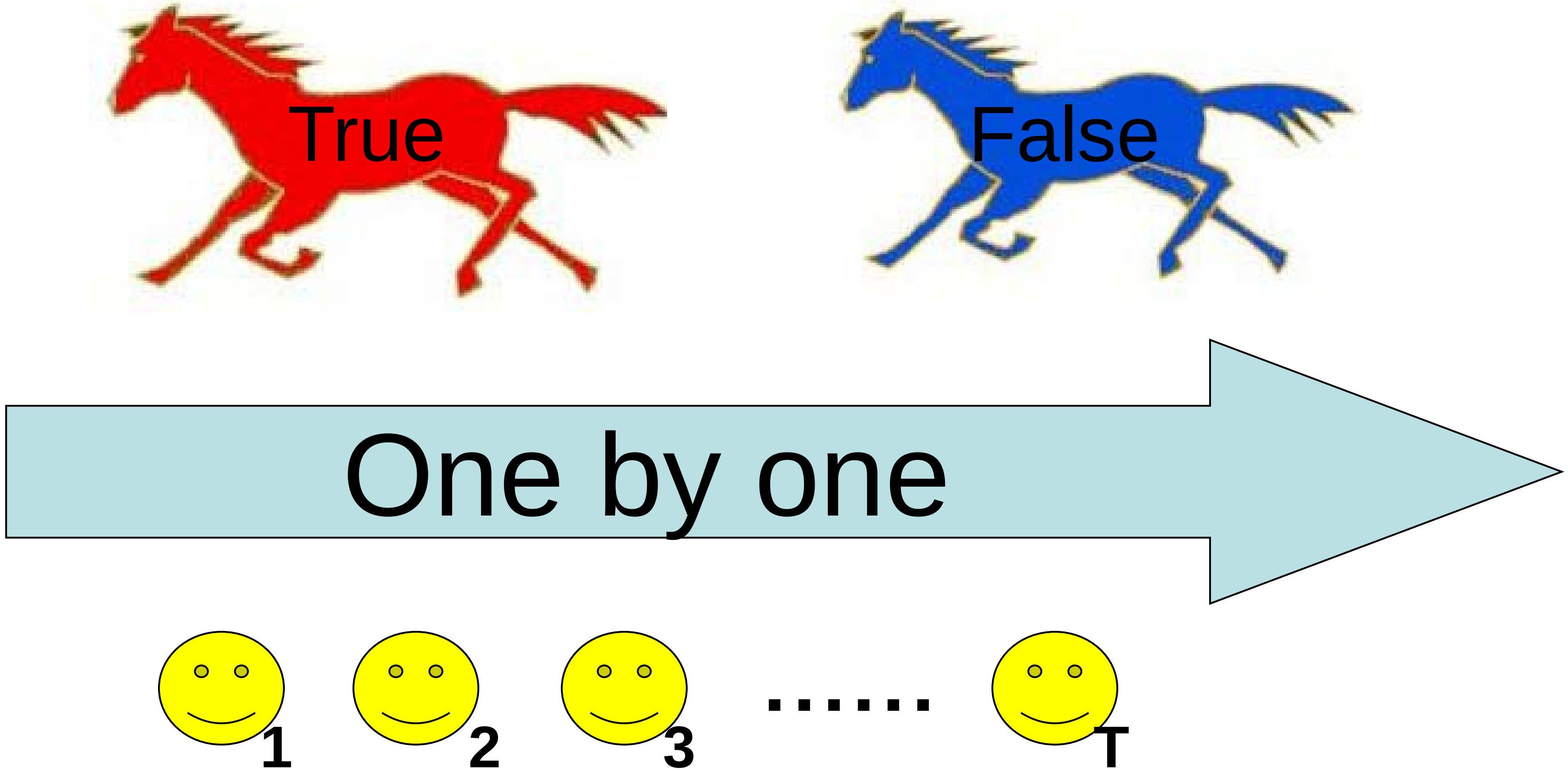
Question

Information Cascade is Fragile or Stable ?

If Stable, Phase Transition ?

Sequential Voting Experiment

Quiz : A or B

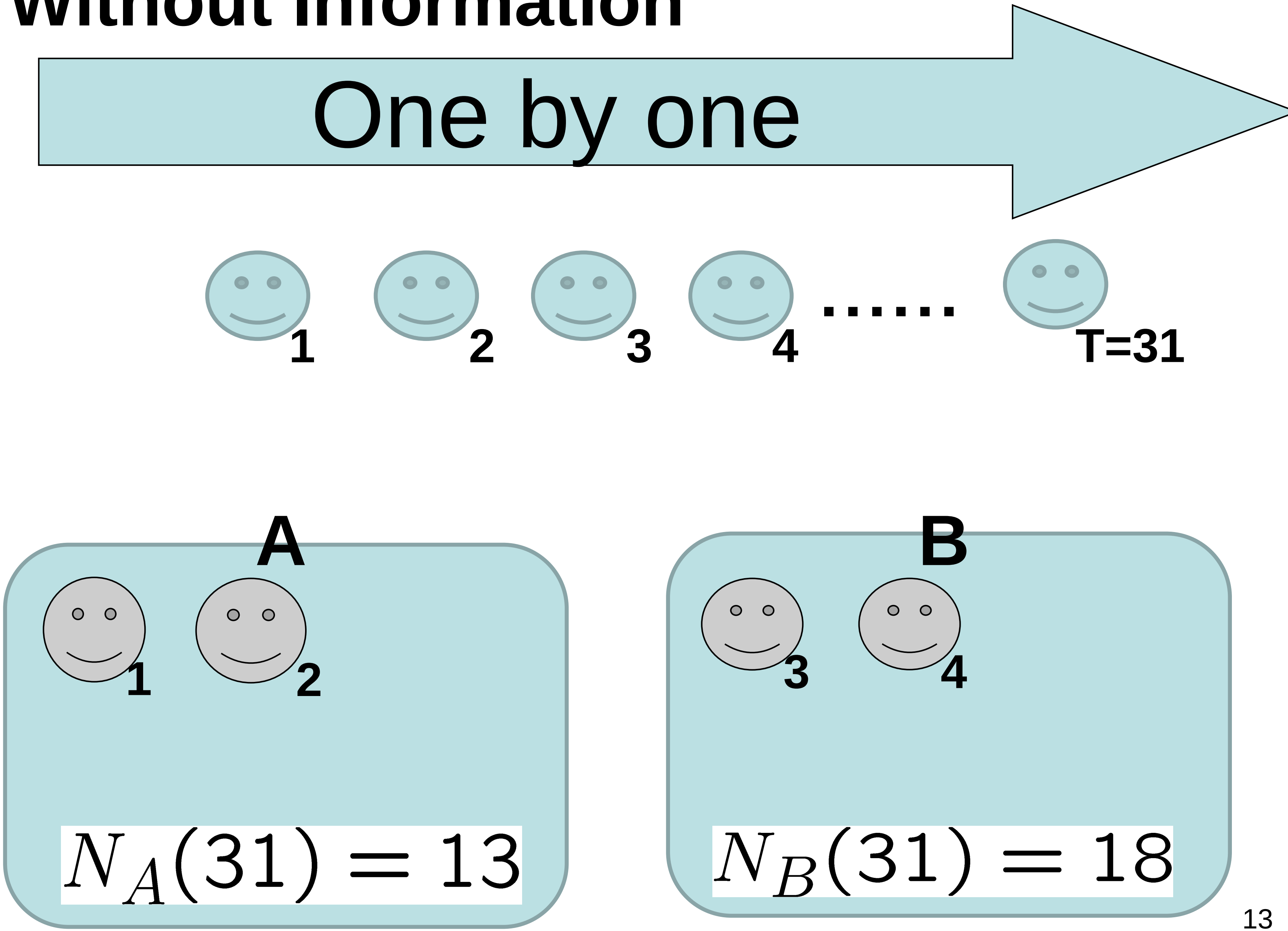


3000 yen + 1000yen for Top 10 students.
31(=T) *2=62 students,100 questions.

14: How much is the price on Buggy the Clown ?
The captain of Buggy's pirates.（at 2010.9.1)

A. 13 millions B
B. 15 millions B

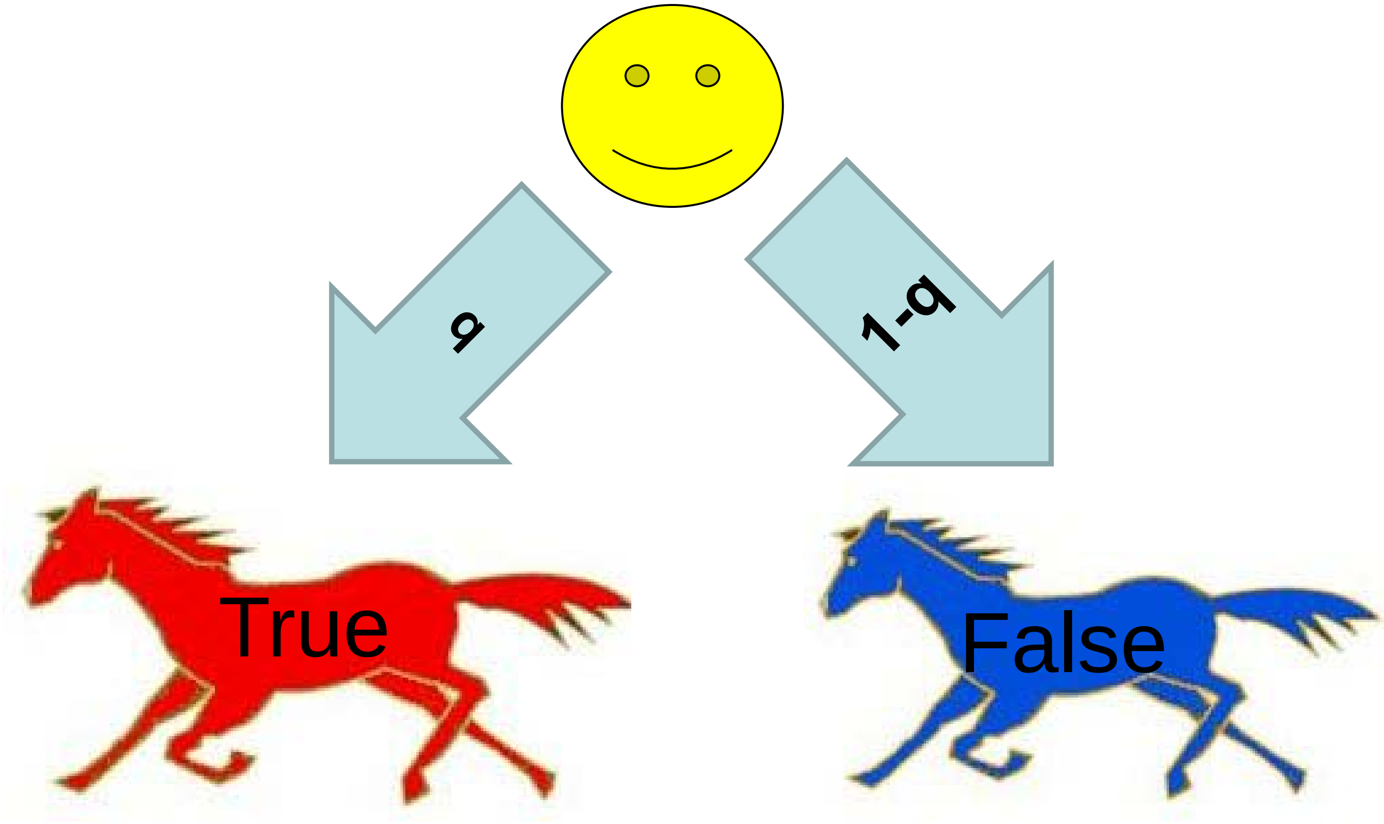
Without Information



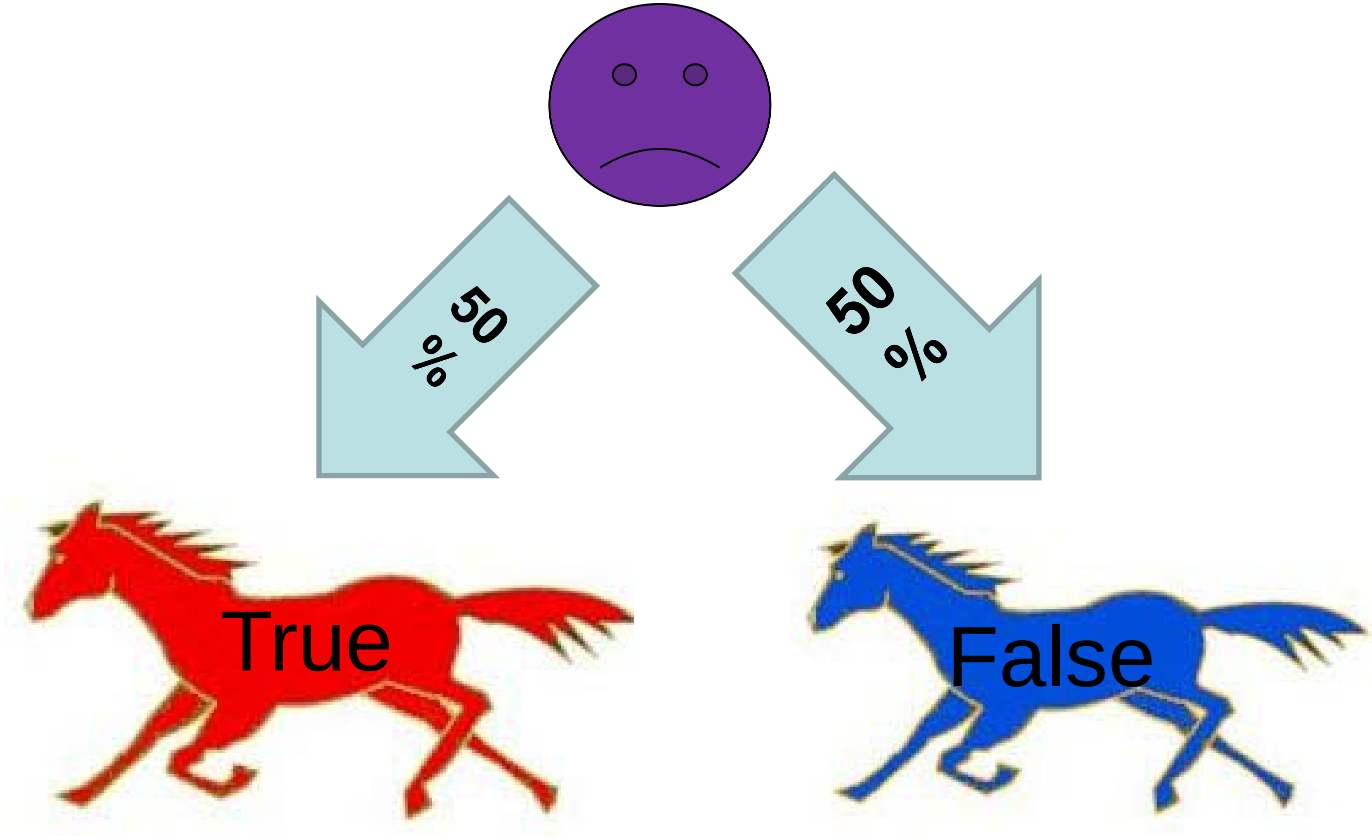
Independent Voter

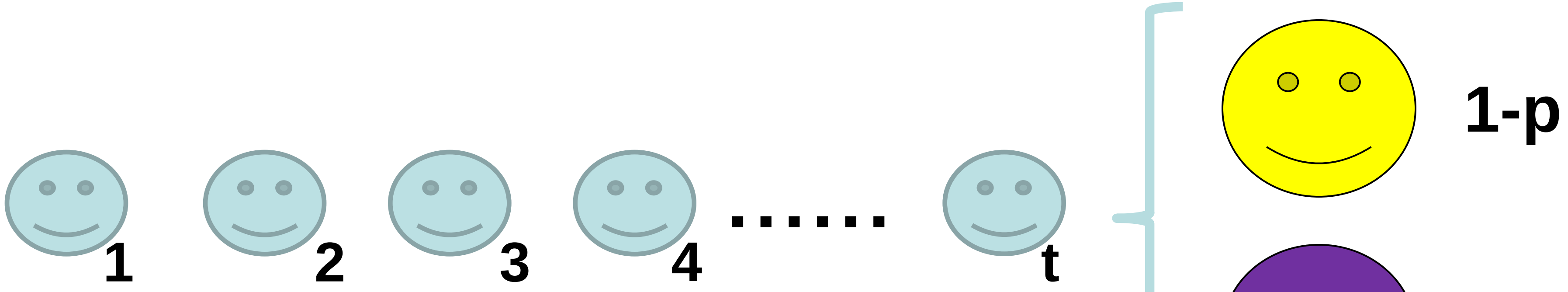
who knows the answer with prob. q=100%.

who is not affected by others' choices.



Stupid Voter (Herder, Herding Voter) who does not know the answer.



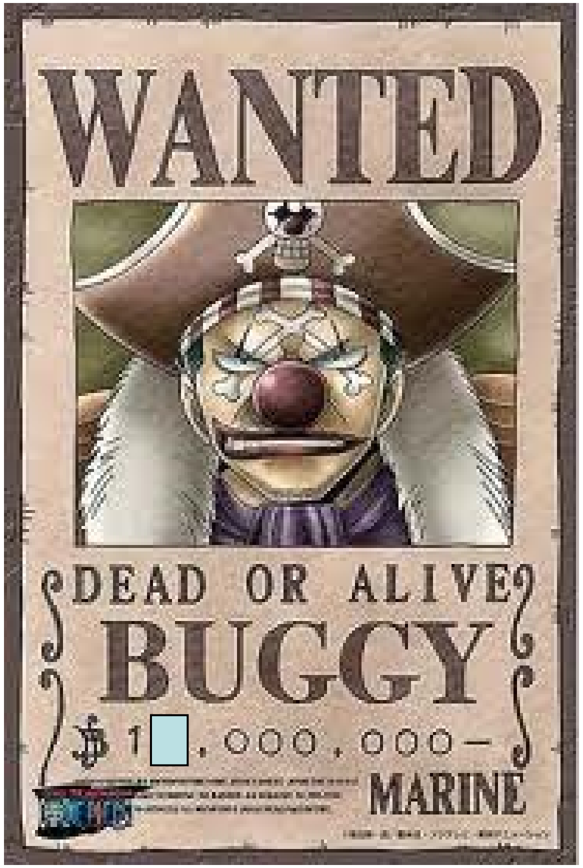


$$\hat{N}_1(t) + \hat{N}_0(t) = t$$

$\hat{N}_1(t)$ $\hat{N}_0(t)$

$$E\left[\frac{\hat{N}_1(t)}{t}\right] = (1-p) + 0.5 \cdot p = 1 - 0.5 \cdot p$$

$$E\left[\frac{\hat{N}_0(t)}{t}\right] = 0.5 \cdot p$$



A. 13 millions

B. 15 millions

Result

$$N_1(31) = 18$$
$$N_0(31) = 13$$

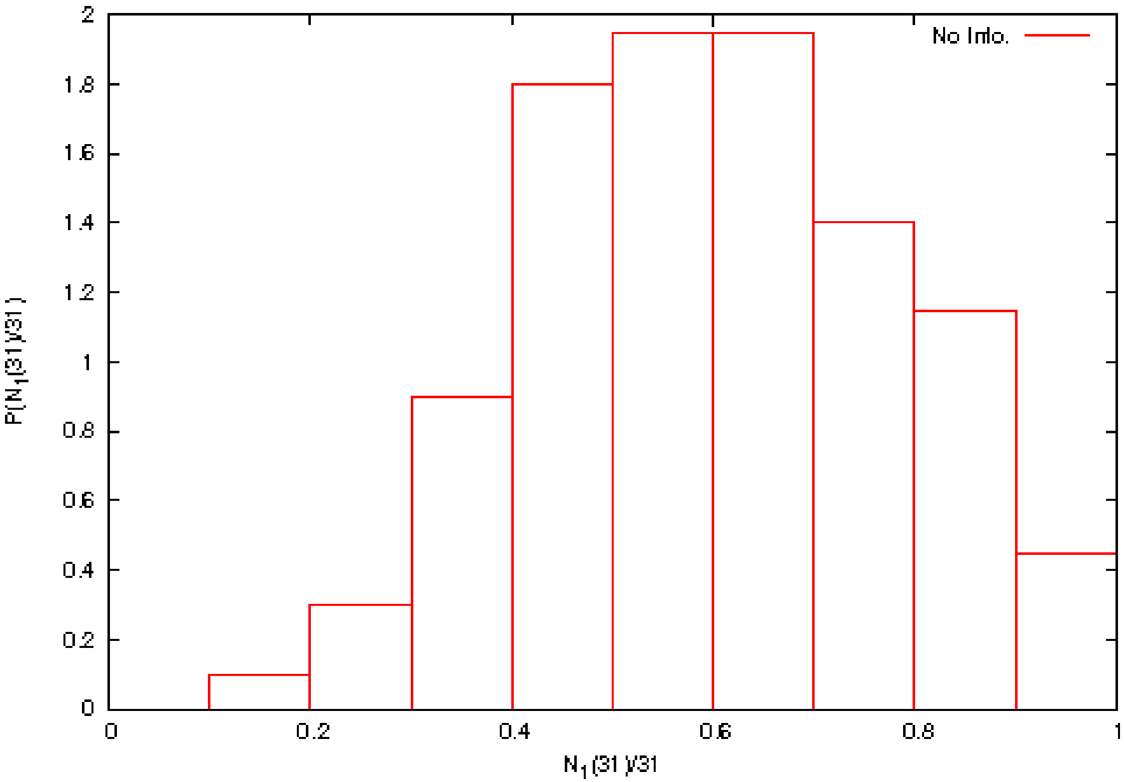
$$1 - \frac{1}{2}p = \frac{18}{31}$$

$$p_{\text{MLE}} = \frac{26}{31}$$

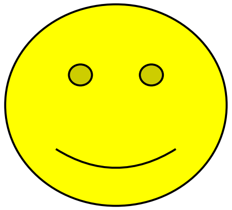
MLE=Most Likelihood Estimate

$$P(\hat{N}_1(T)/T)$$

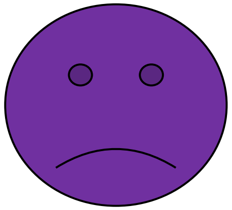
$T = 31,200$quiz



$$E\left[\frac{\hat{N}_1(T)}{T}\right] = 1 - 0.5 \cdot p \simeq 0.6 \Rightarrow p \simeq 0.8$$

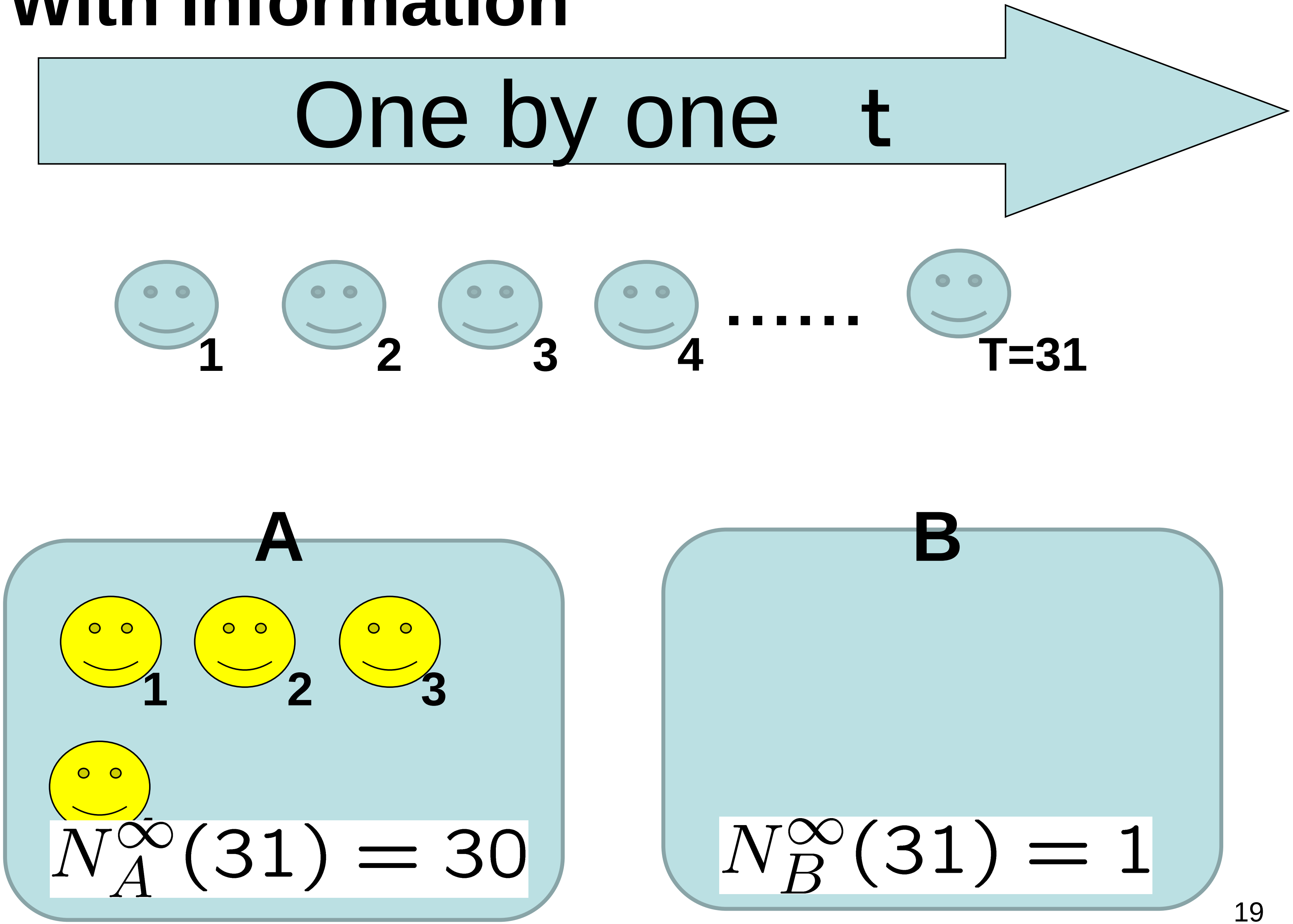


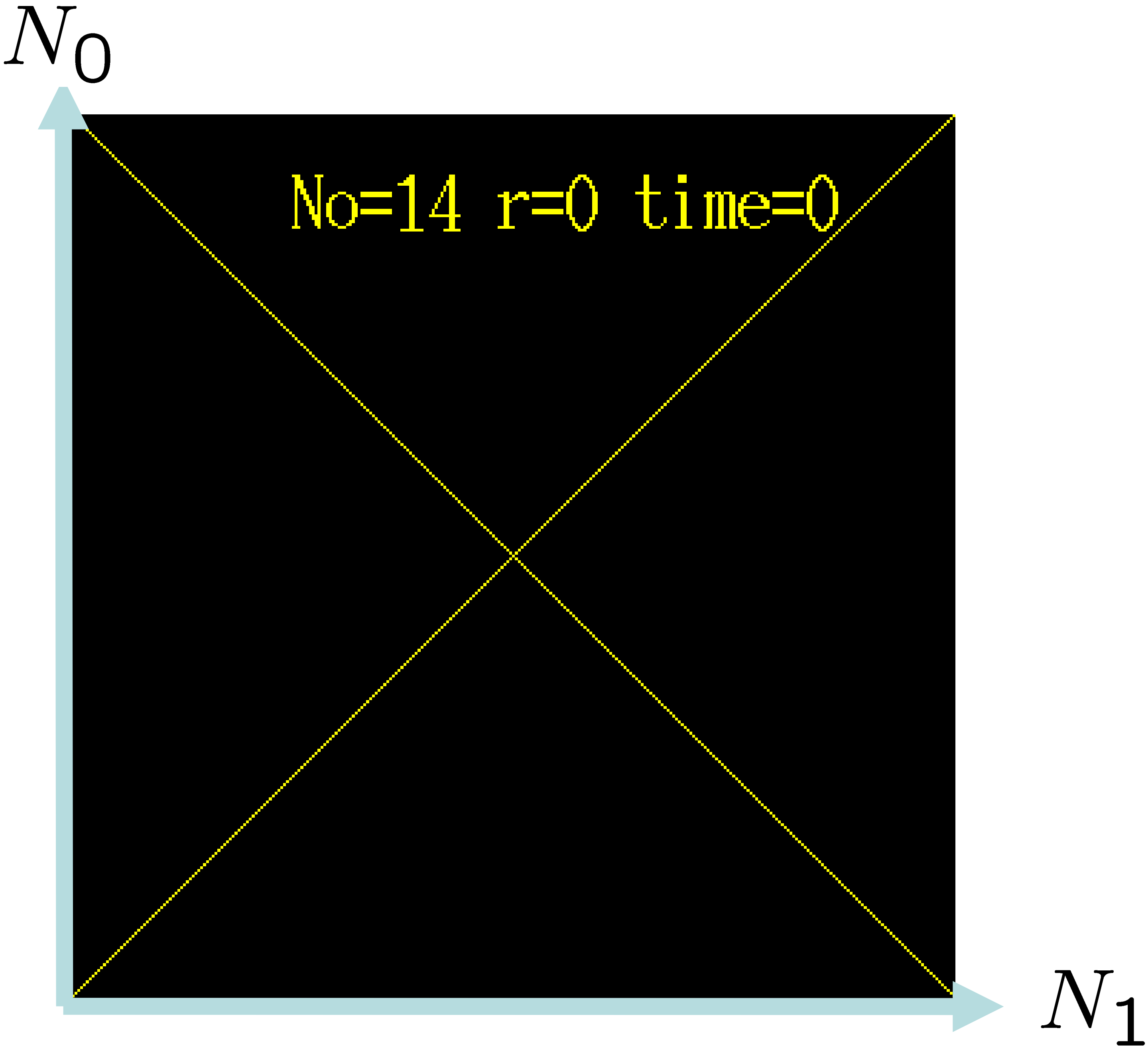
20%



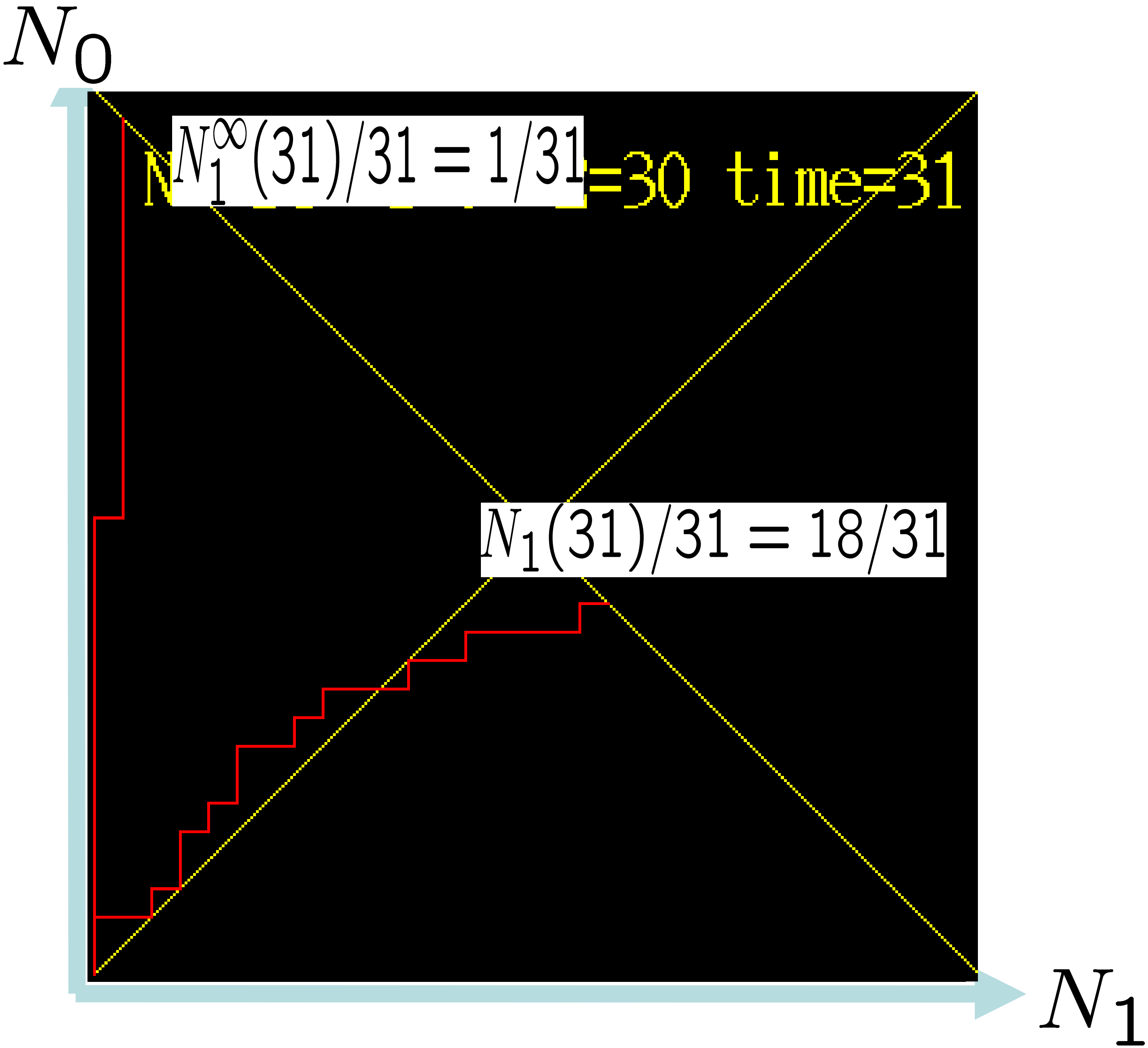
80%

With Information



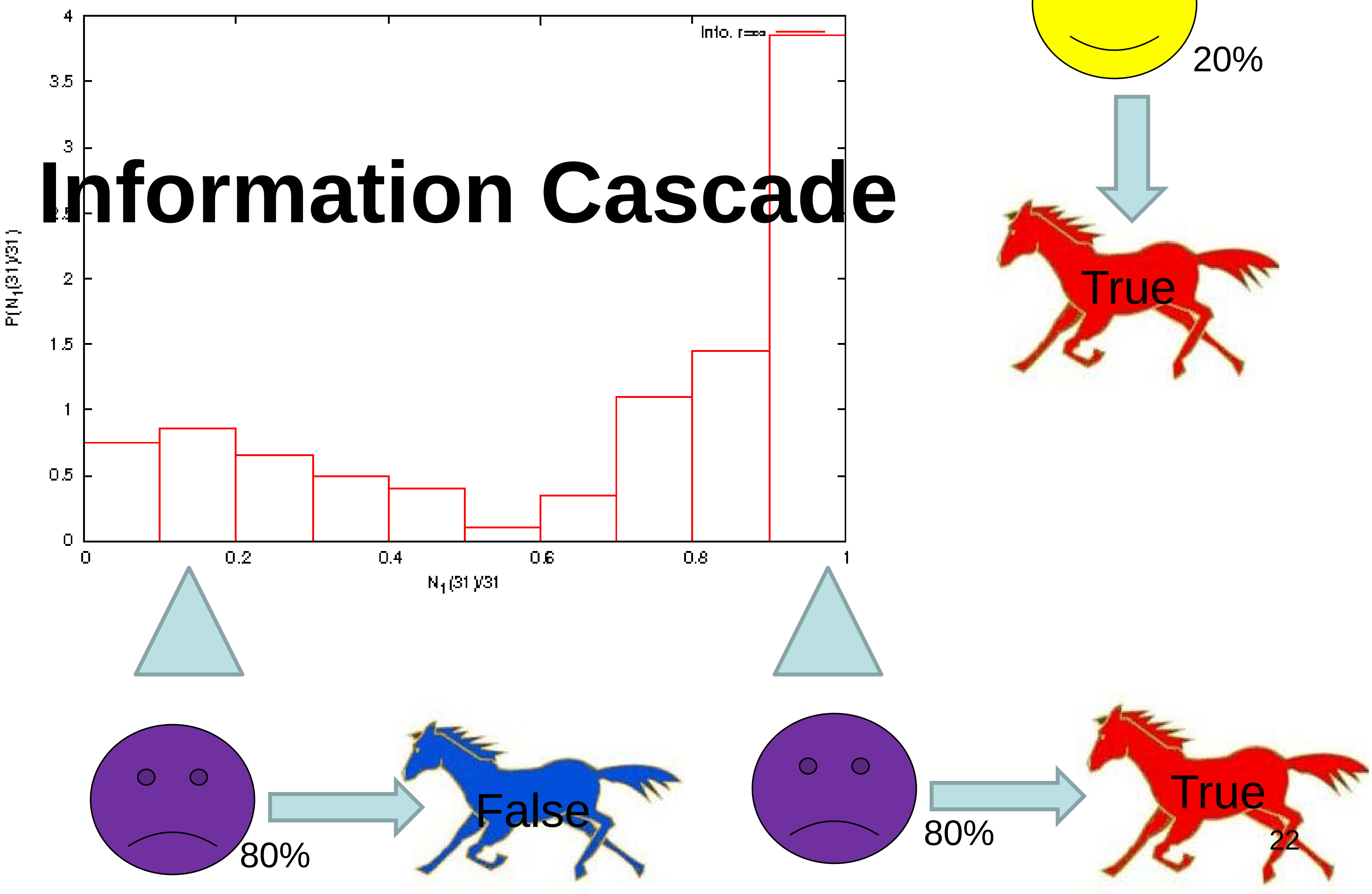


N_0
No=14 r=0 time=0
N_1


N_0
$N_1^{\infty}(31)/31 = 1/31$
=30 time=31
$N_1(31)/31 = 18/31$
N_1

$$P(N_1^{\infty}(T)/T)$$

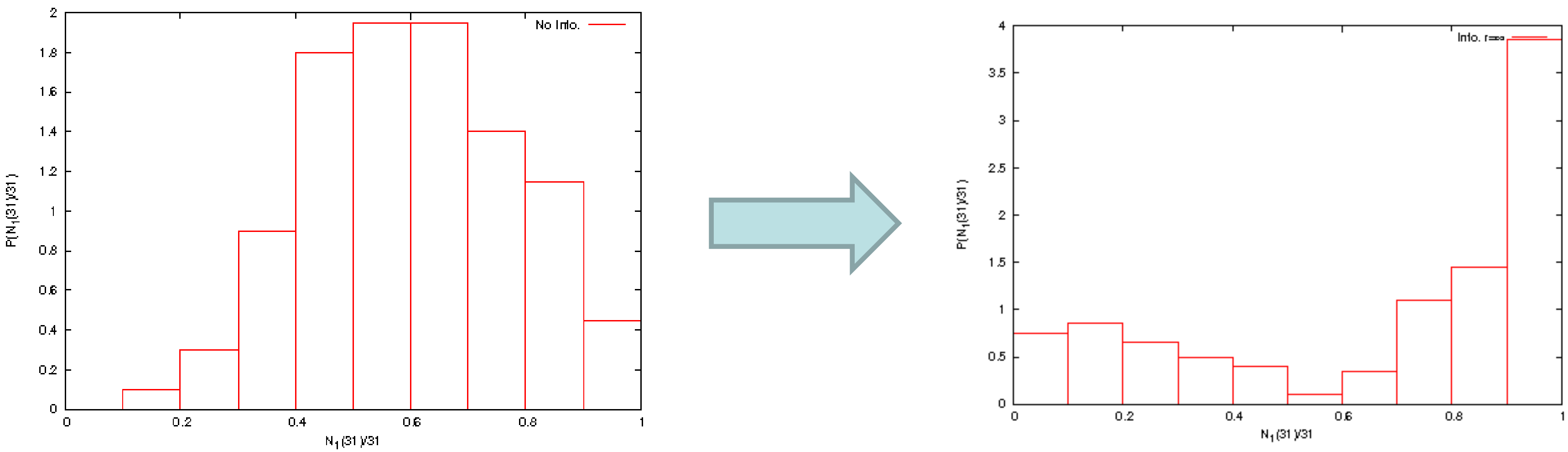
Information Cascade



Question

What is the Physics

in the Information Cascade ?



Digital herders and phase transition in a voting model
M.Hisakado and S.Mori,J.Phys.A,Math.Theor.44(2011)275204

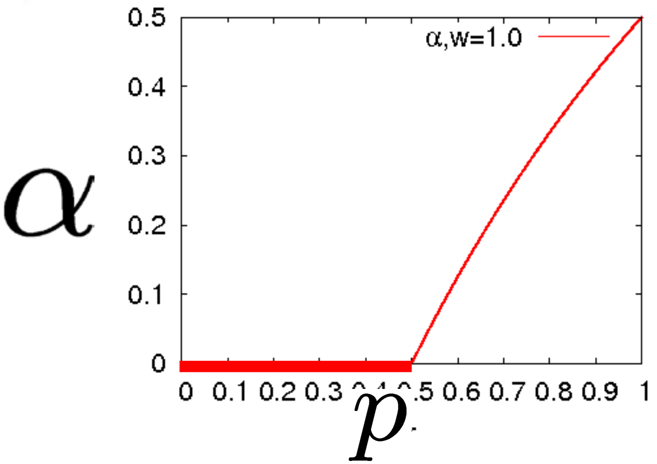
Conclusion

The Information Cascade is
a second-order phase transition.

If $p > p_c$ the Prob. that 
condensate to 

becomes >0

α



Voting Model

Exact Scale Invariance in Mixing of Binary Candidates in Voting Model
S. Mori and M. Hisakado, J.Phys.Soc.Jpn,79,vol.3(2010)034001-034008.

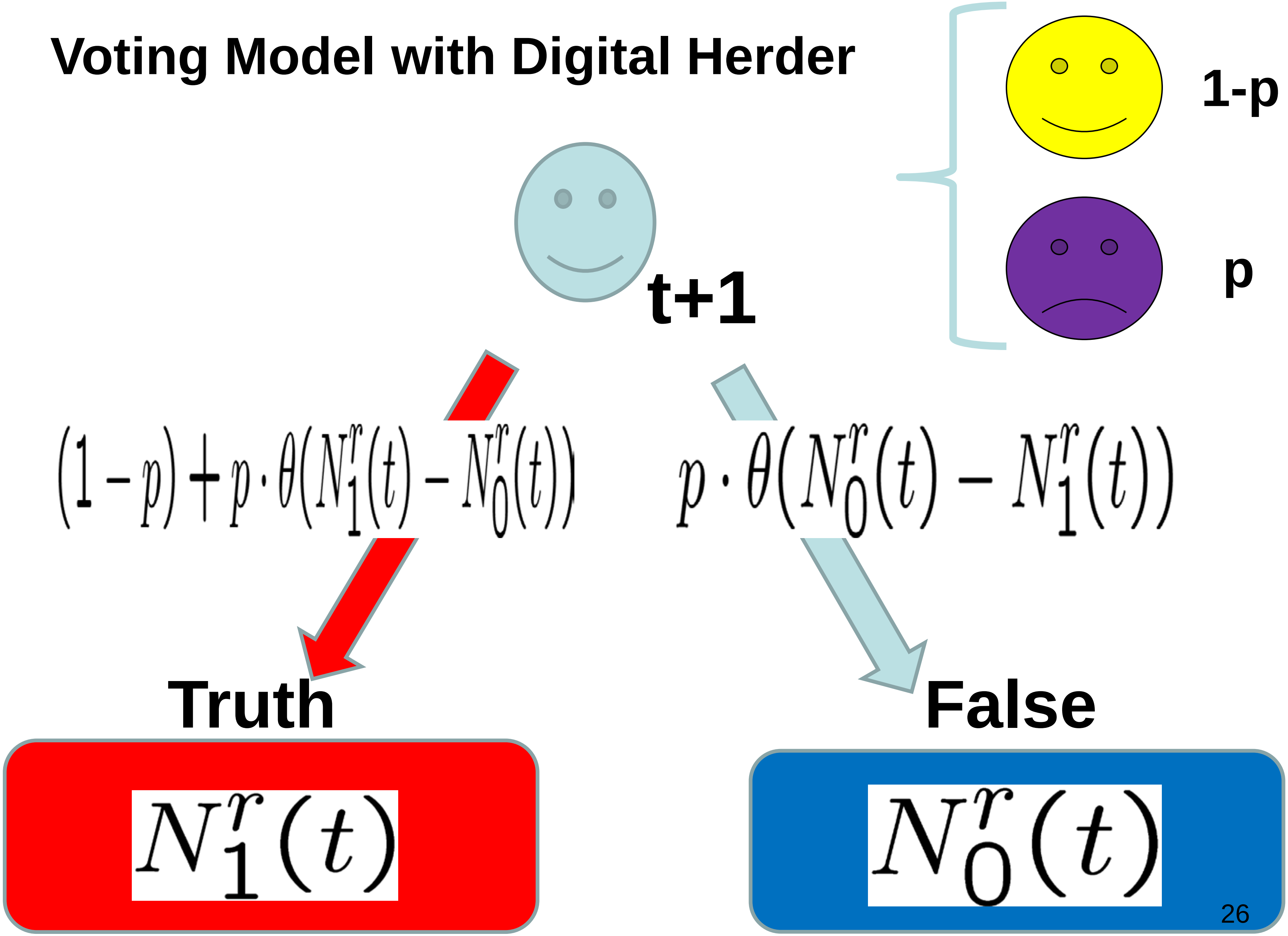
Phase transition and information cascade in voting model
M. Hisakado and S.Mori, J.Phys.A,Math.Theor.43(2010)315207.

Component Ratios of Independent and Herding Betters in a Racetrack Betting Market, S.Mori and M.Hisakado, preprint arXiv:1006.4884.

Horse Race Betting

Digital herders and phase transition in a voting model
M.Hisakado and S.Mori,J.Phys.A,Math.Theor.44(2011)275204

Voting Model with Digital Herder



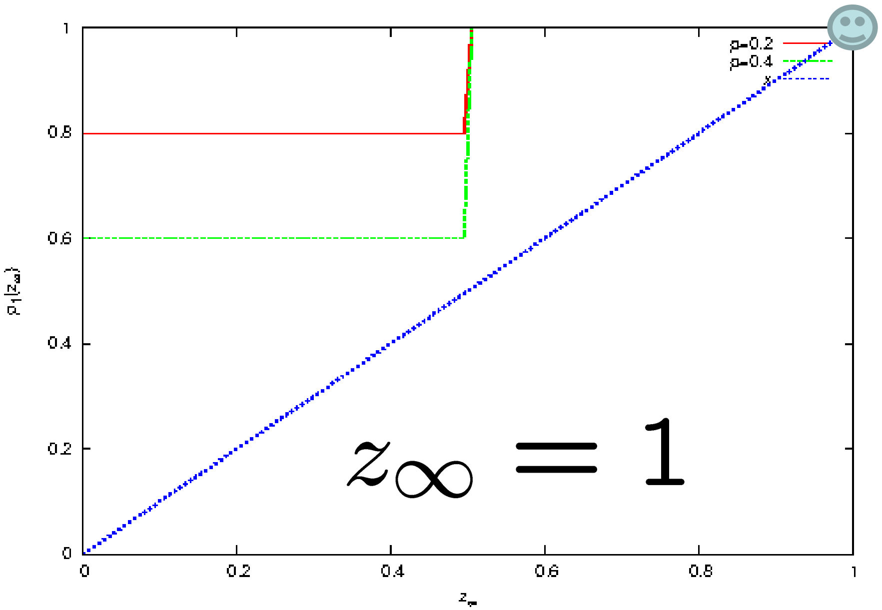
Mean Field Eq. $r = \infty, t \to \infty$

$$\lim_{t \to \infty} \frac{N_1^{\infty}(t)}{t} = z_{\infty}$$

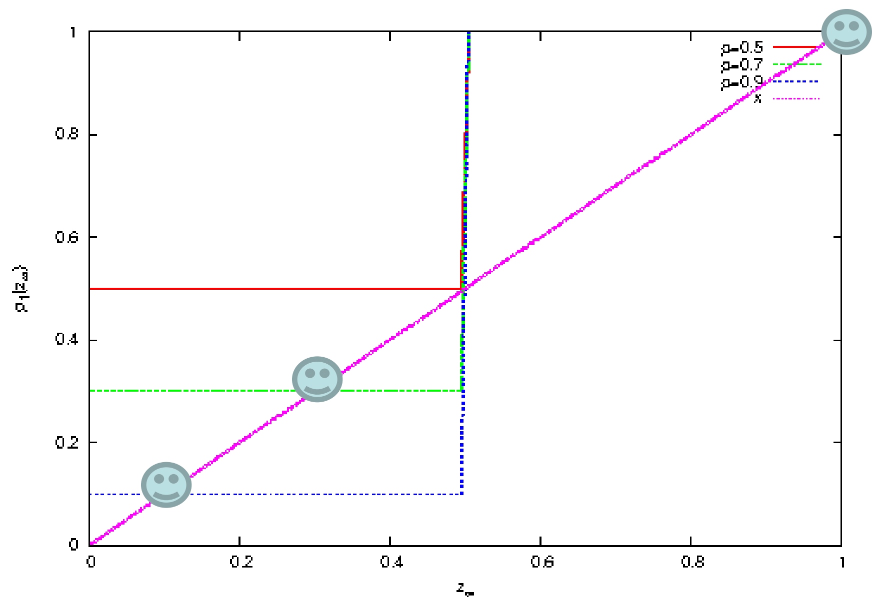
$$z_{\infty} = (1 - p) + p\theta(z_{\infty} - \frac{1}{2})$$

$$p < p_c = \frac{1}{2}$$

$$p \geq p_c$$



$$z_{\infty} = 1$$



$$z_{\infty} = 1 - p \qquad z_{\infty} = 1$$

Mean Field Eq.

$$\text{Prob}(\hat{X}_{t+1}=1)=(1-p)+p\theta(\sum_{s=1}^{t}\hat{X}_s-\frac{1}{2}t)$$

$$z_\infty \equiv \lim_{t\to\infty}\text{Prob}(\hat{X}_t=1)=\lim_{t\to\infty}E(\hat{X}_t)$$

$$\text{Prob}(\sum_{s=1}^{t}\hat{X}_s=n)\equiv P(t,n)={}_tC_n\cdot z_\infty^n(1-z_\infty)^{t-n}$$

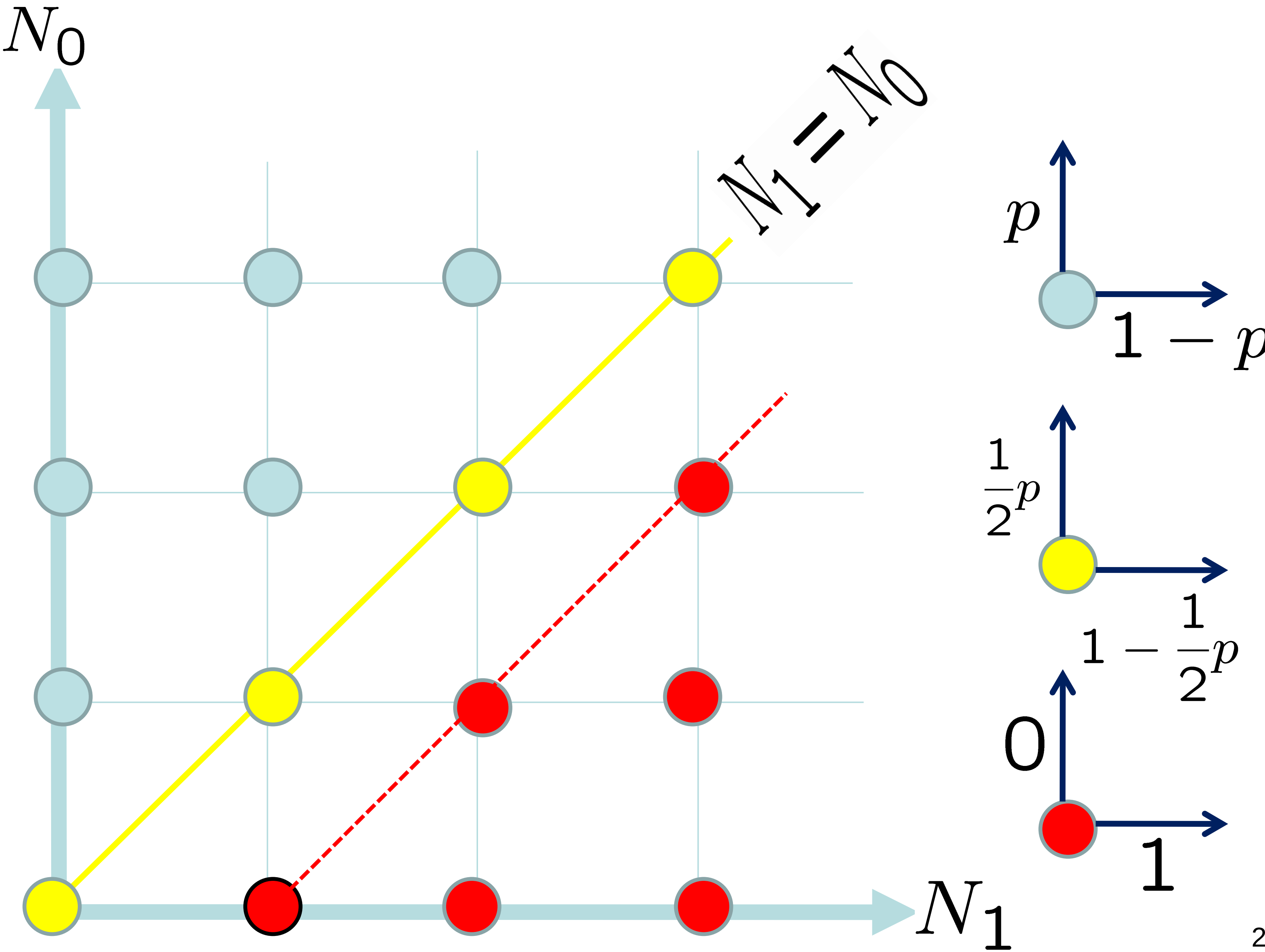
Self-Consistent eq.

$$E(\text{Prob}(\hat{X}_{t+1}=1))=(1-p)+p\cdot\sum_{n=1}^{t}P(t,n)\theta(n-\frac{1}{2}t)$$

$$t\to\infty(r=\infty)$$

$$\lim_{t\to\infty}P(t,n)=\lim_{t\to\infty}{}_tC_n z_\infty^n(1-z_\infty)^{t-n}=\delta_{n,t\cdot z_\infty}$$

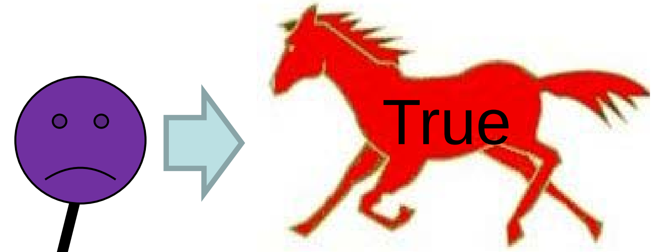
$$z_\infty=(1-p)+p\theta(z_\infty-\frac{1}{2})$$


N_0
$N_1 = N_0$
N_1
p
$1-p$
$\frac{1}{2}p$
$1-\frac{1}{2}p$
0
1

1:Truth

$$N_1^{\infty}(t)$$

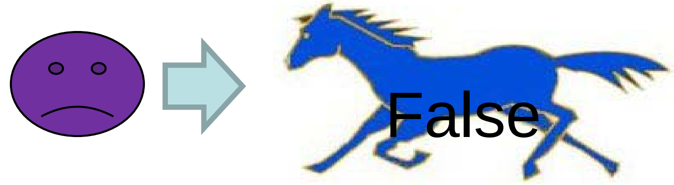
All



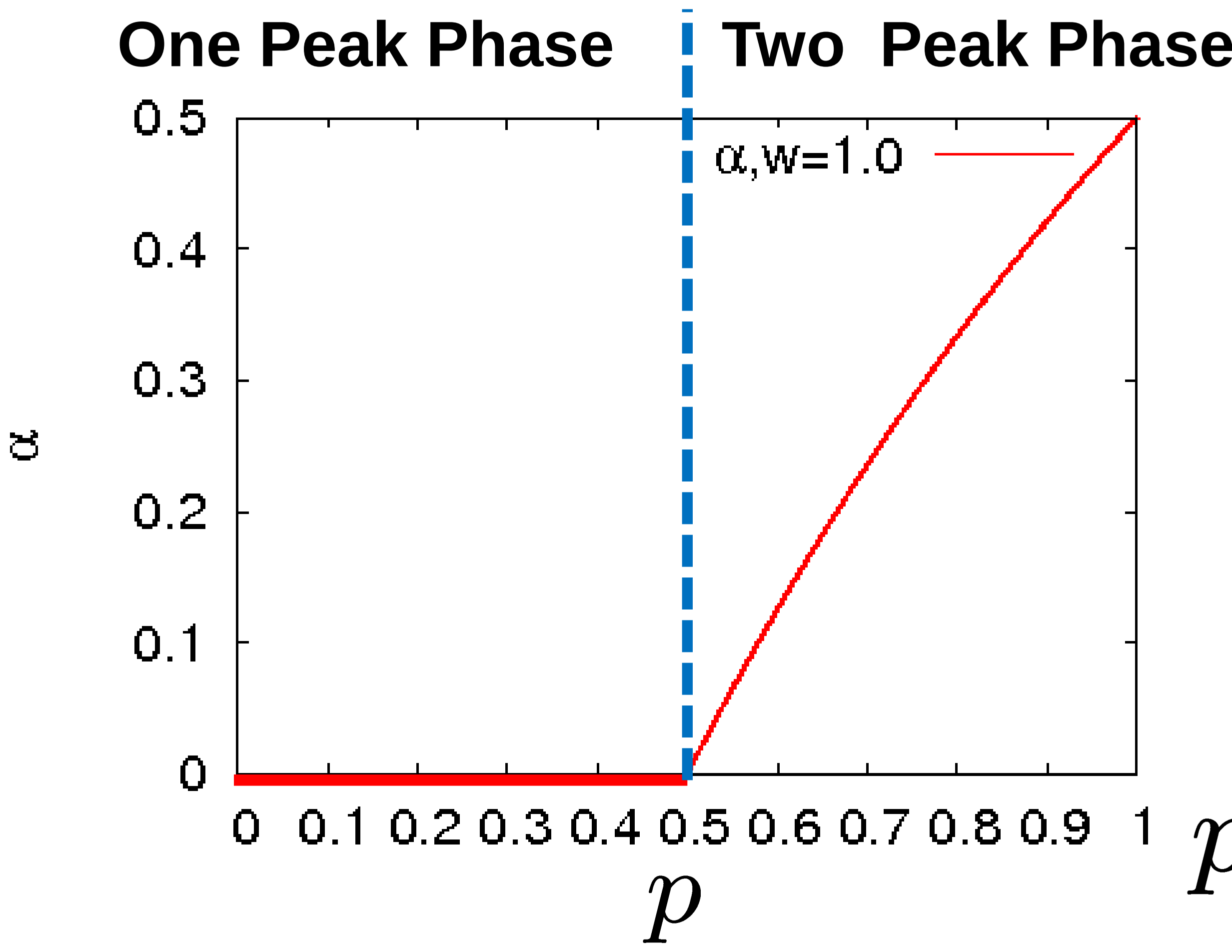
$$\hat{Z}(t) \equiv \frac{\hat{N}_1^{\infty}(t)}{t} \sim \alpha\delta_{1-p} + (1-\alpha)\delta_1, \ t \to \infty$$

All

$$\alpha = \frac{2p - 1 + |2p-1|}{3 + |2p-1|}$$

$\alpha = \text{Prob.}$ **(All** 

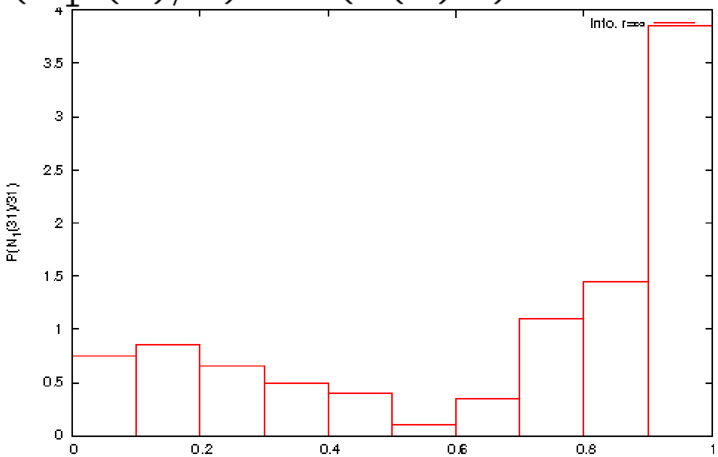
)



$$p_c = \frac{1}{2}$$

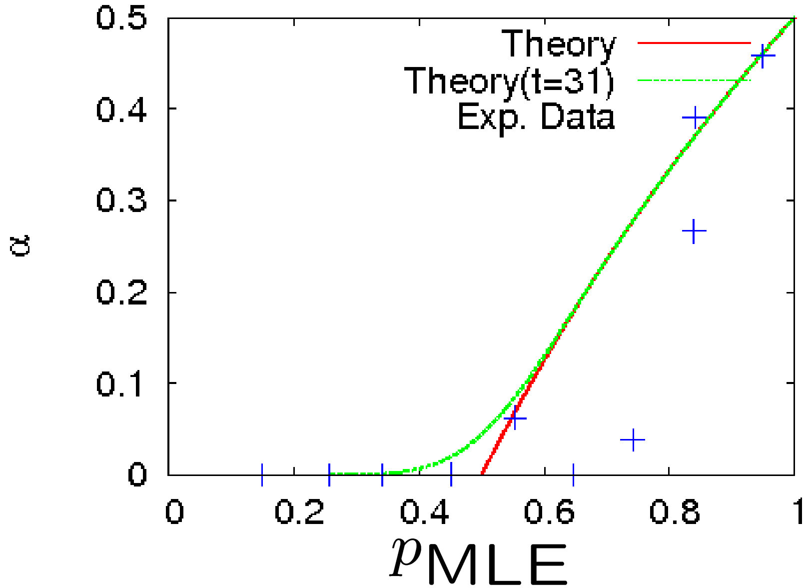
Theory vs Experiment

$$P(N_1^{\infty}(T)/T) = P(Z(T)^{\infty})$$



$$\lim_{t\to\infty} \widehat{Z}(t) = \alpha\delta_{1-p} + (1-\alpha)\delta_1$$

$$p = 0.8$$

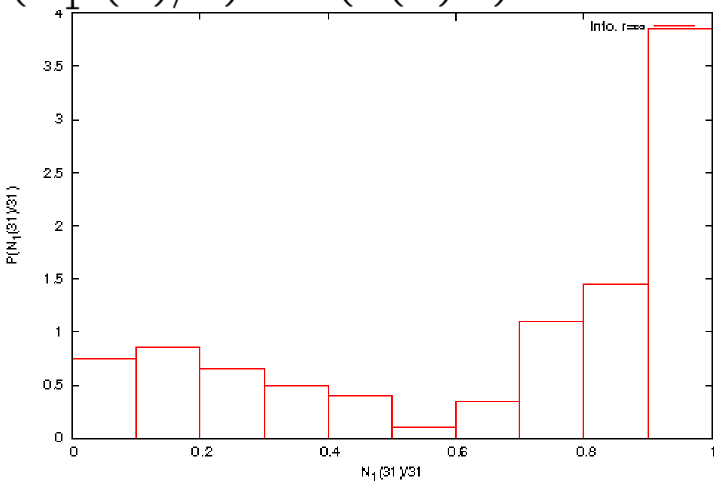


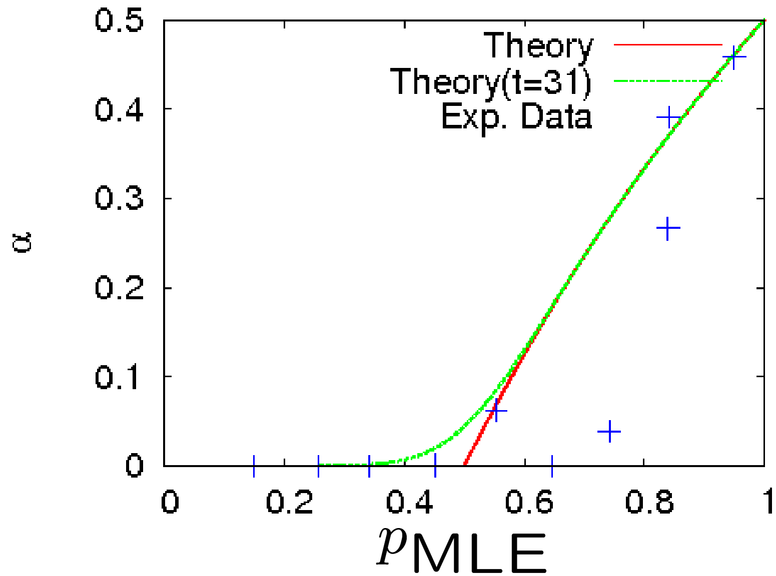
$$\alpha = \frac{2p-1+|2p-1|}{3+|2p-1|}$$

Conclusion 1

The Information Cascade is
a second-order phase transition.

$P(N_1^{\infty}(T)/T) = P(Z(T)^{\infty})$





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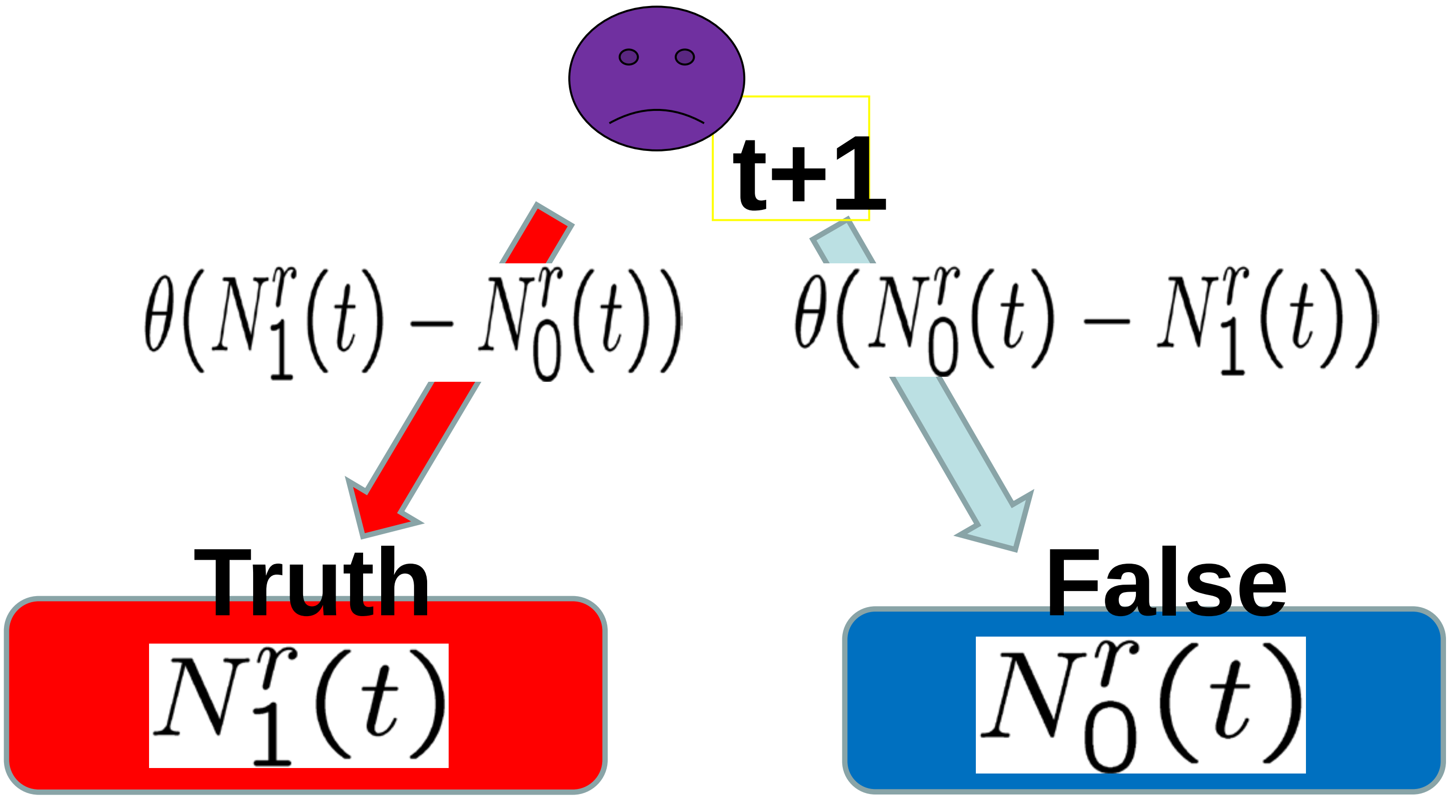
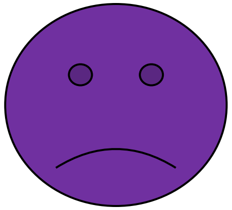
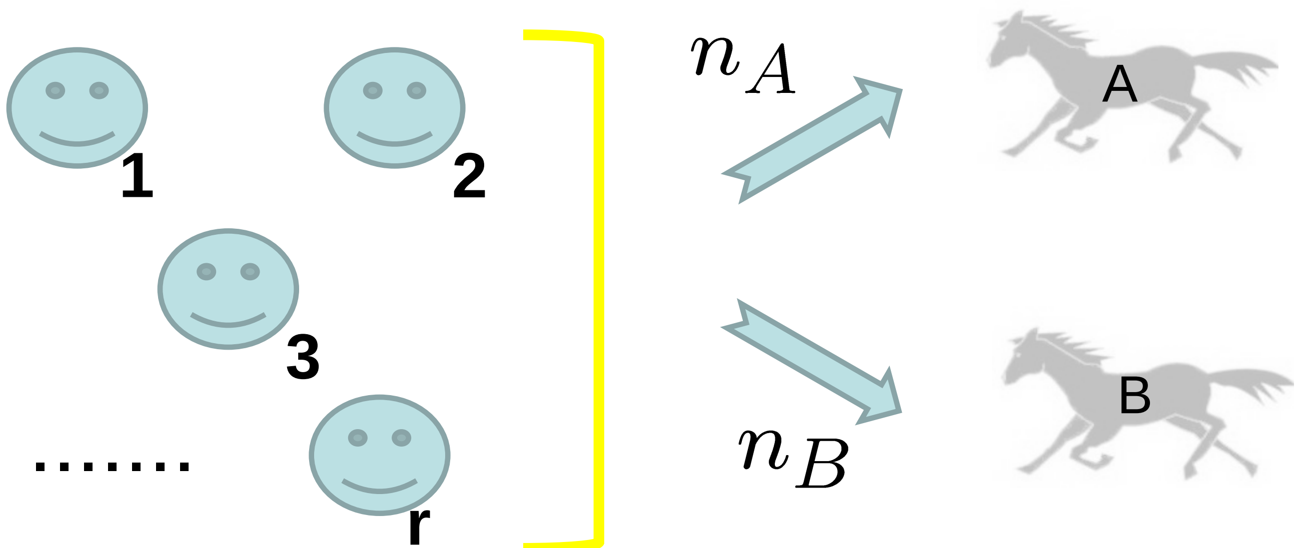
He votes as digital herder ?



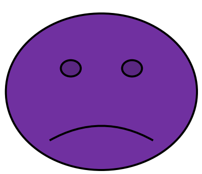
How he should votes ?



r voters



$$\Pr(\widehat{X}_i = 1) = q$$



How he estimate ?

$$\Pr(A = \text{True} | n_A = n), \text{Prob}(B = \text{True} | n_B = r - n)$$

Answer from Bayes' theorem

$$\mathsf{Pr}(A = \mathsf{True}|n_A = n) = \frac{1}{2}(\tanh(\lambda(n-(r-n)))+1)$$

$$\lambda = \log\sqrt{\frac{q}{1-q}}$$

If $|n_A - n_B| >> 0$ or $q >> \frac{1}{2}$

$$\mathsf{Pr}(A = \mathsf{True}|n_A) = \theta(n_A - n_B)$$

$$T \to \infty$$

If  are independent,

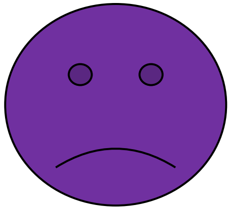
$$\text{Pr(Majority=True)} = 1$$

If  are independent+digital herder,

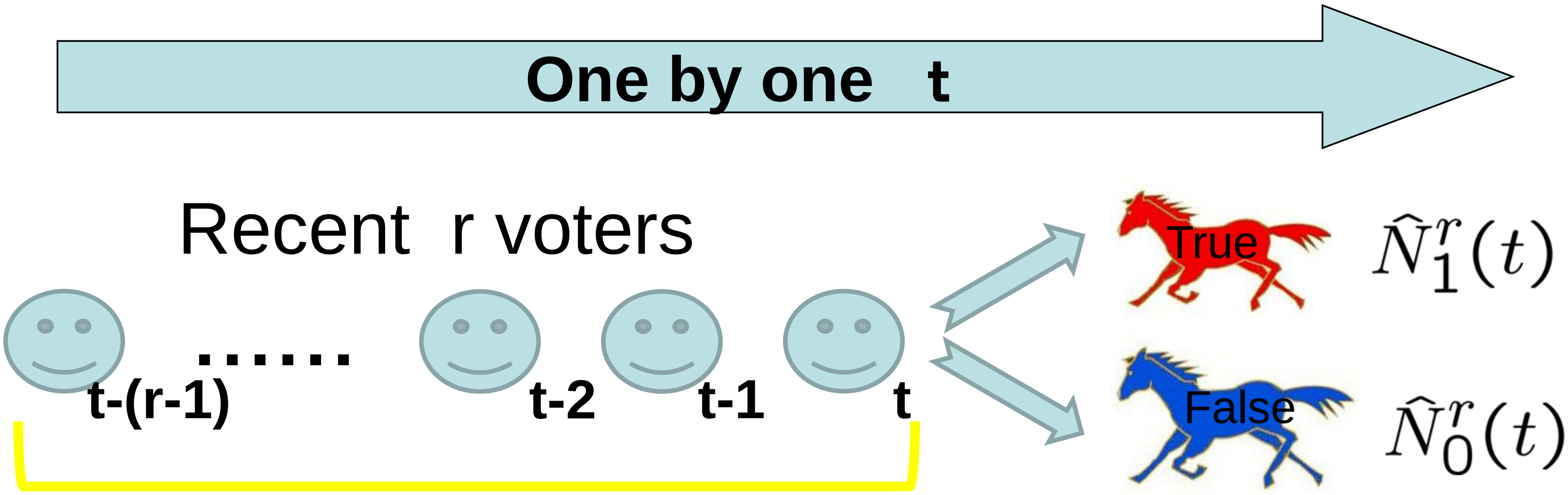
$$\text{Pr(Majority=True)} = 1 - \alpha$$

$$\alpha = \frac{2p - 1 + |2p - 1|}{3 + |2p - 1|}$$

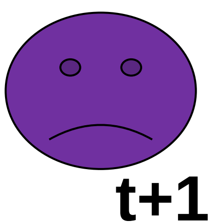
How he votes ?



Control of information transmission

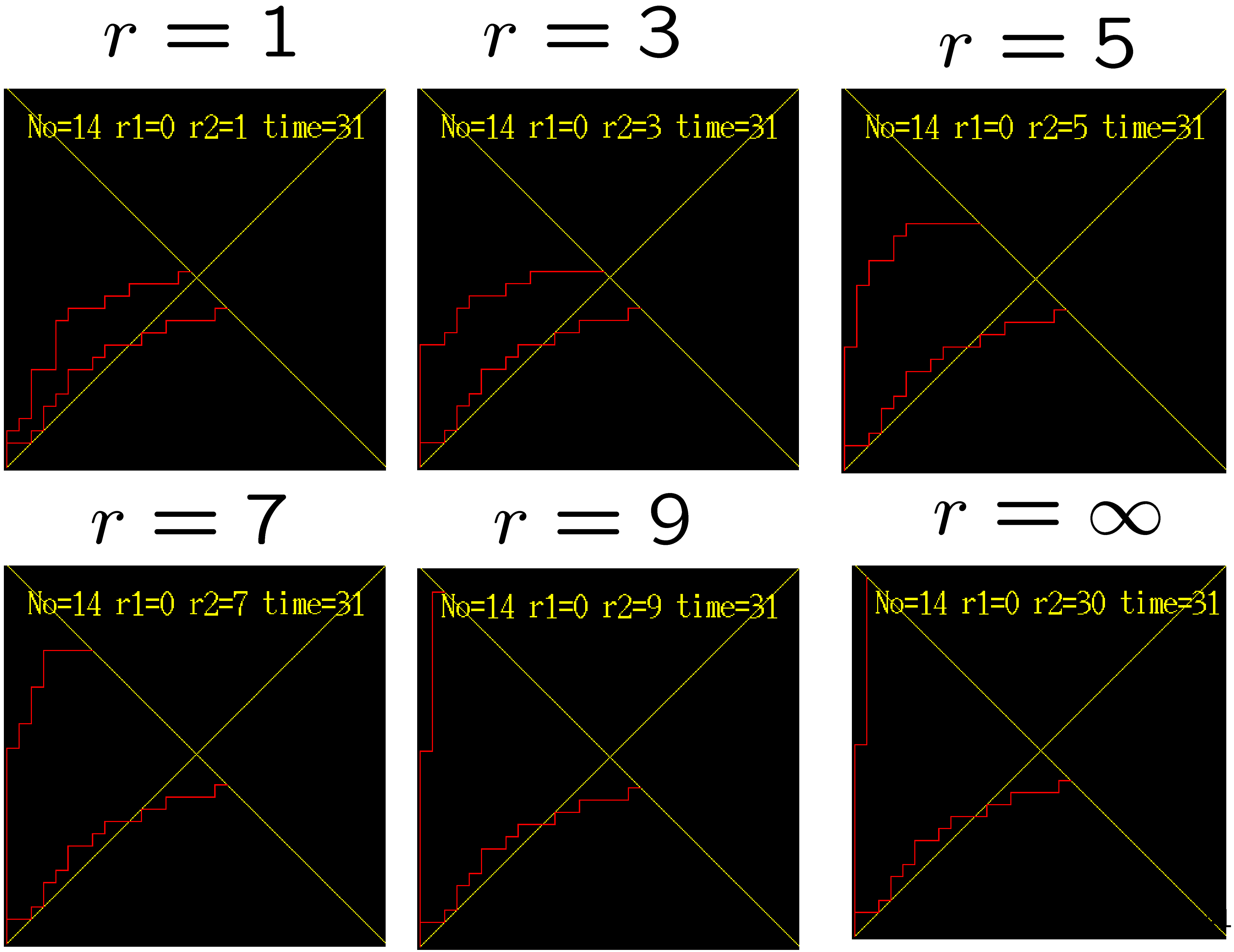


$$\hat{N}_1^r(t) + \hat{N}_0^r(t) = r$$



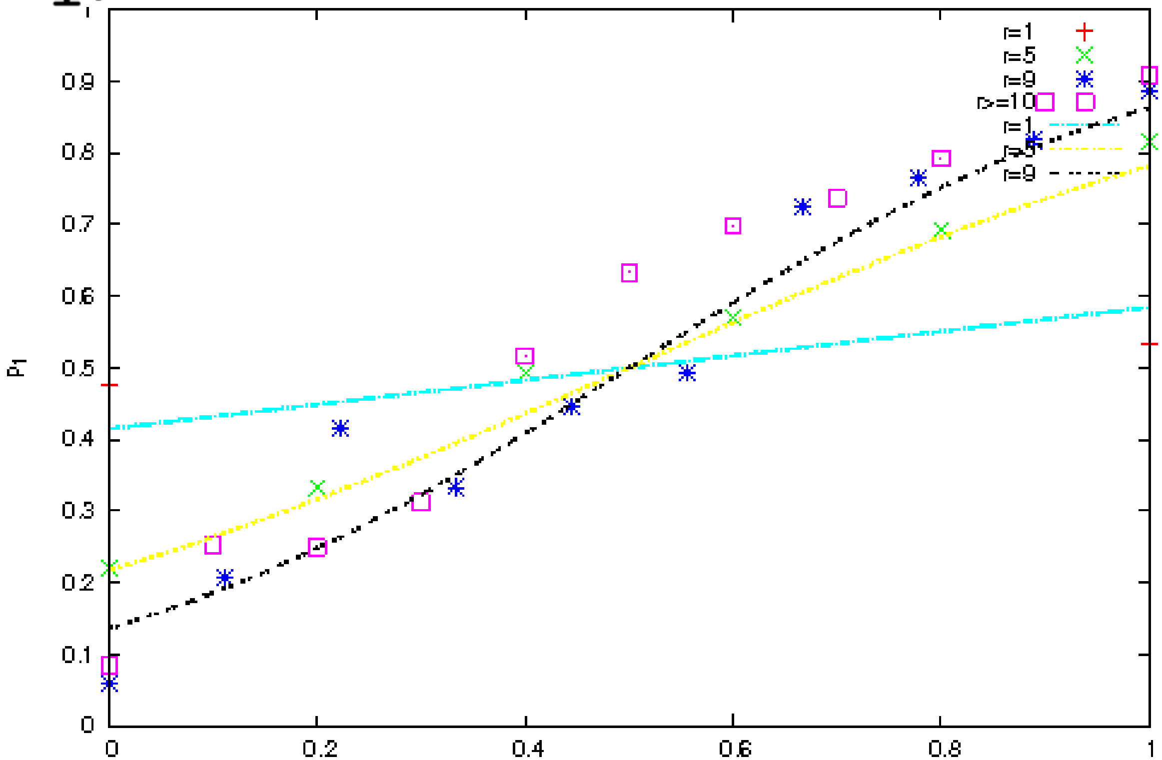
How he votes ?

$$p_1(r, N_1^r)$$



$r = 1$
No=14 r1=0 r2=1 time=31
$r = 3$
No=14 r1=0 r2=3 time=31
$r = 5$
No=14 r1=0 r2=5 time=31
$r = 7$
No=14 r1=0 r2=7 time=31
$r = 9$
No=14 r1=0 r2=9 time=31
$r = \infty$
No=14 r1=0 r2=30 time=31

$p_1(r, N_1^r)$



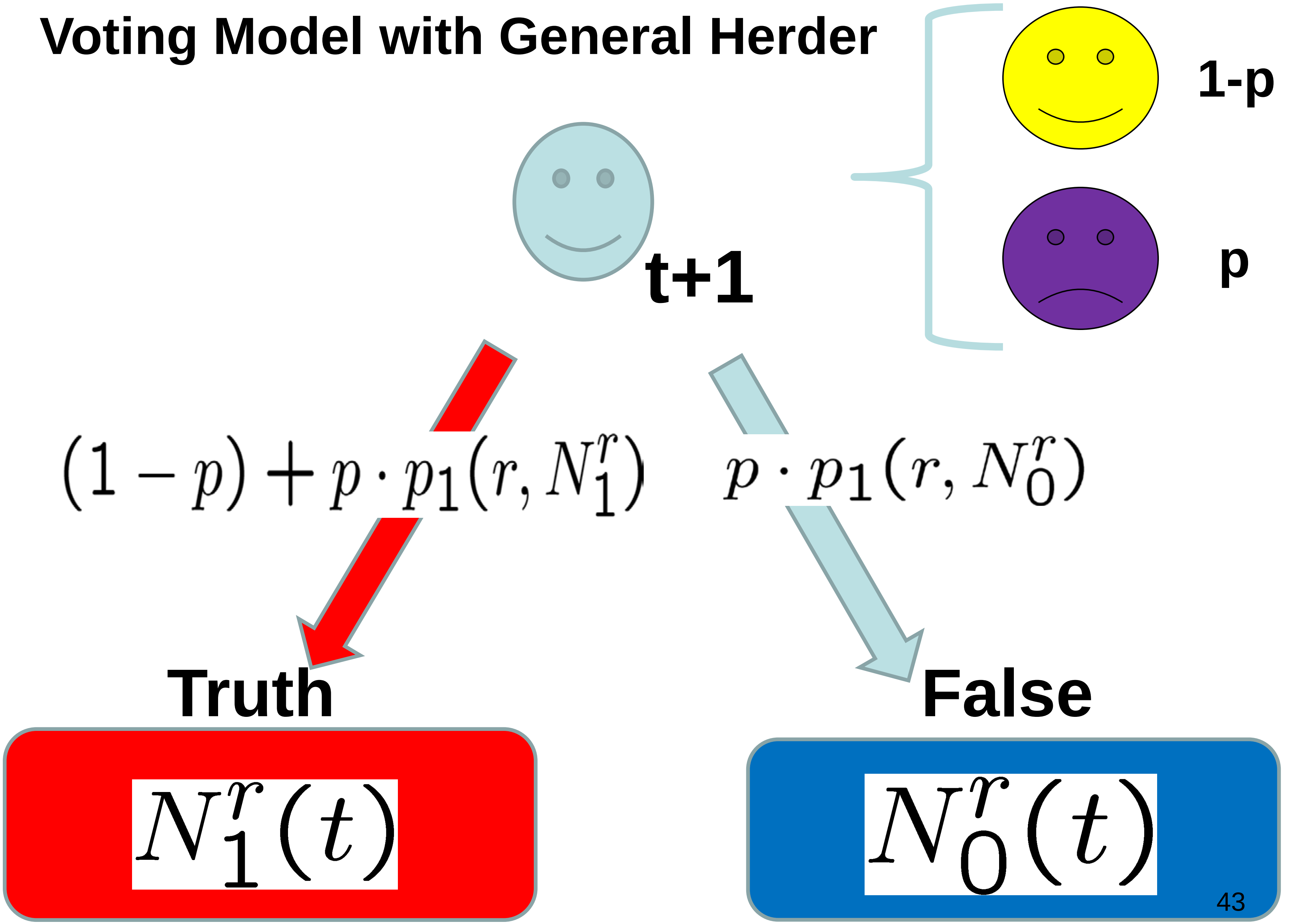
$\dfrac{N_1^r}{r}$

$$p_1(r, N_1^r) = \frac{1}{2}(\tanh \lambda \left(\frac{N_1^r - \frac{1}{2}r}{r + z} \right) + 1)$$

$$\lambda_{MLE} = 4.15, \quad 3.57 \le \lambda \le 4.92 (95\% \text{Conf.})$$

$$z_{MLE} = 11.21, \quad 8.72 \le z \le 14.64 (95\% \text{Conf.})$$

Voting Model with General Herder

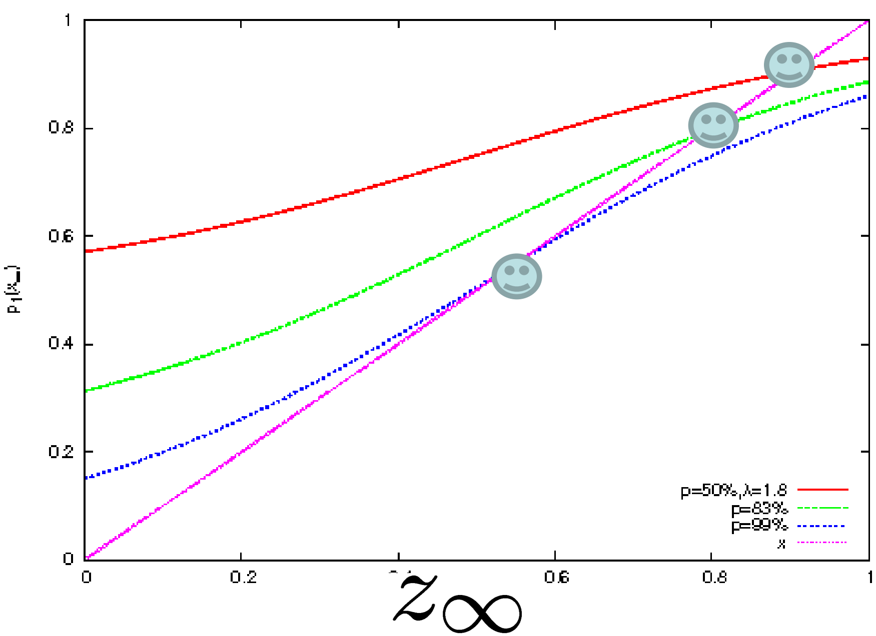


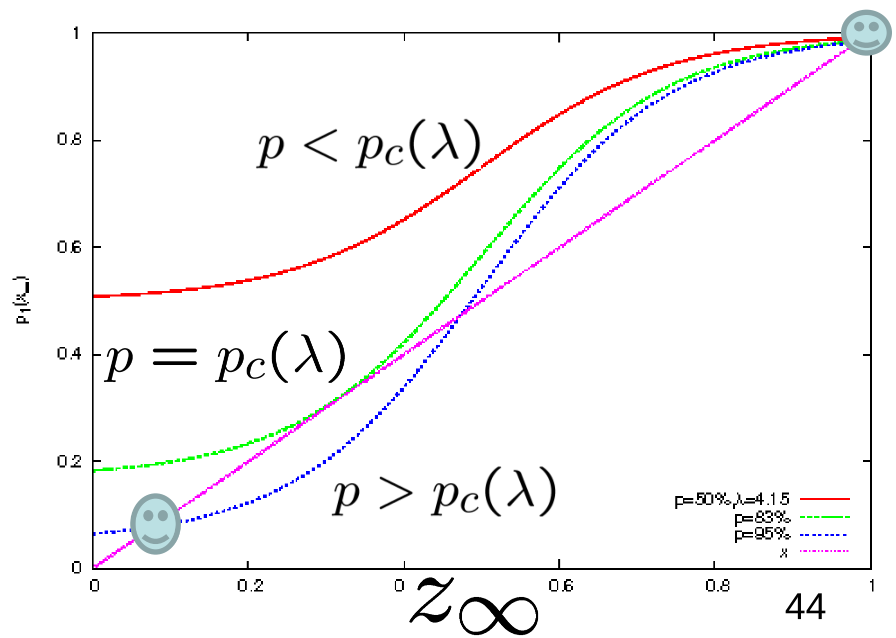
Mean Field Eq. $r = \infty, t \to \infty$

$$\lim_{t \to \infty} \frac{N_1^{\infty}(t)}{t} = z_{\infty}$$

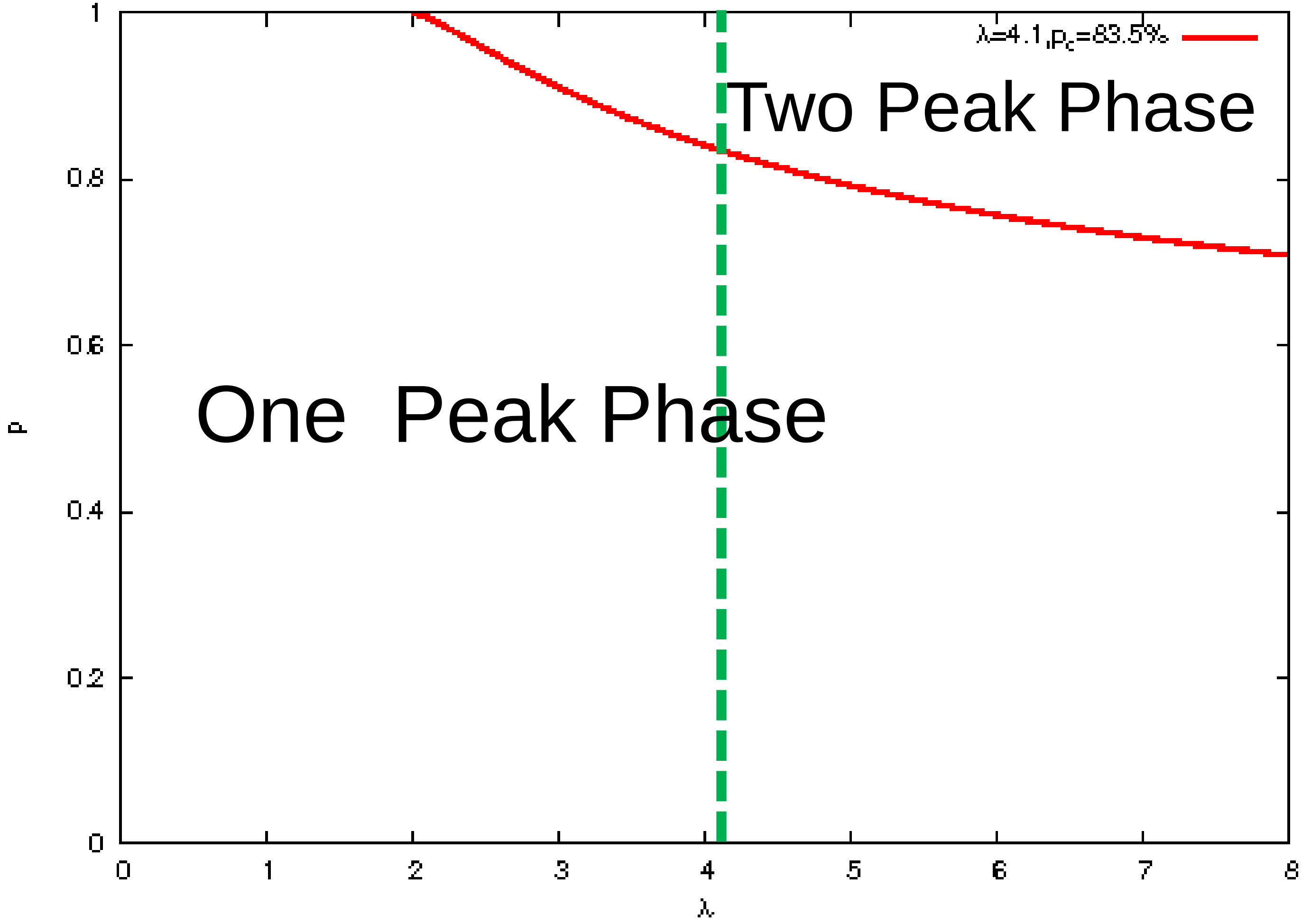
$$z_{\infty} = (1 - p) + p\frac{1}{2}(\tanh \lambda(z_{\infty} - \frac{1}{2}) + 1)$$

$\lambda < \lambda_c = 2$ $\lambda > \lambda_c = 2$

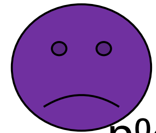
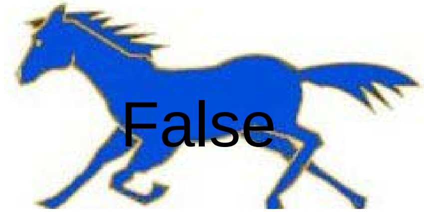




Phase Diagram



$$\lambda_{MLE} = 4.15$$

α = Prob. **(All**  p% → 

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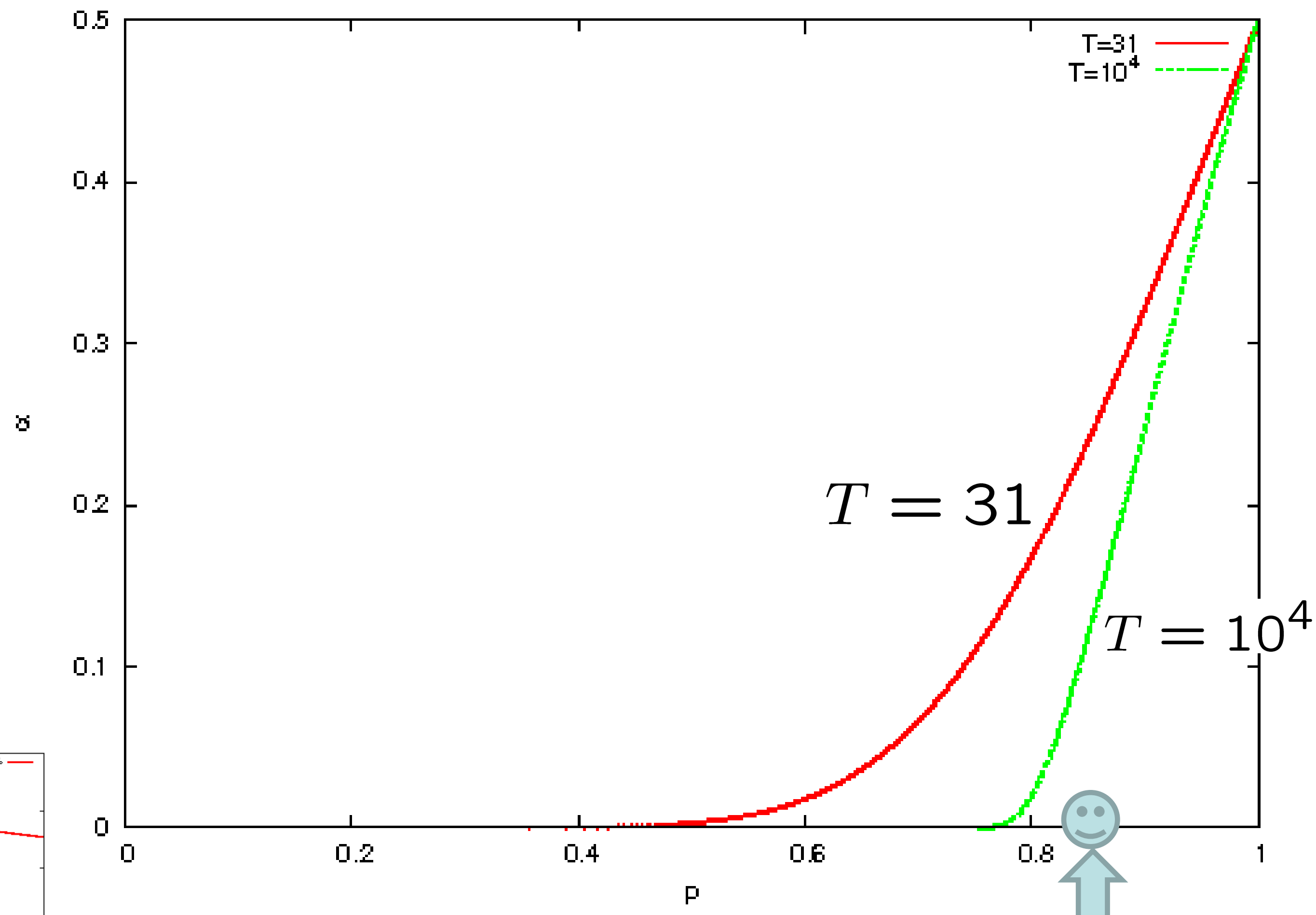
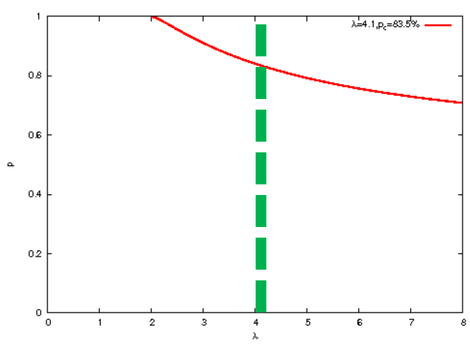
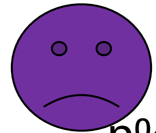
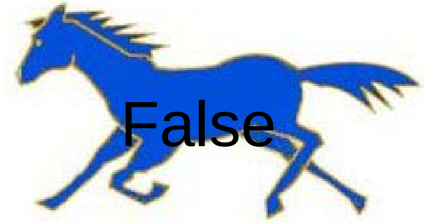
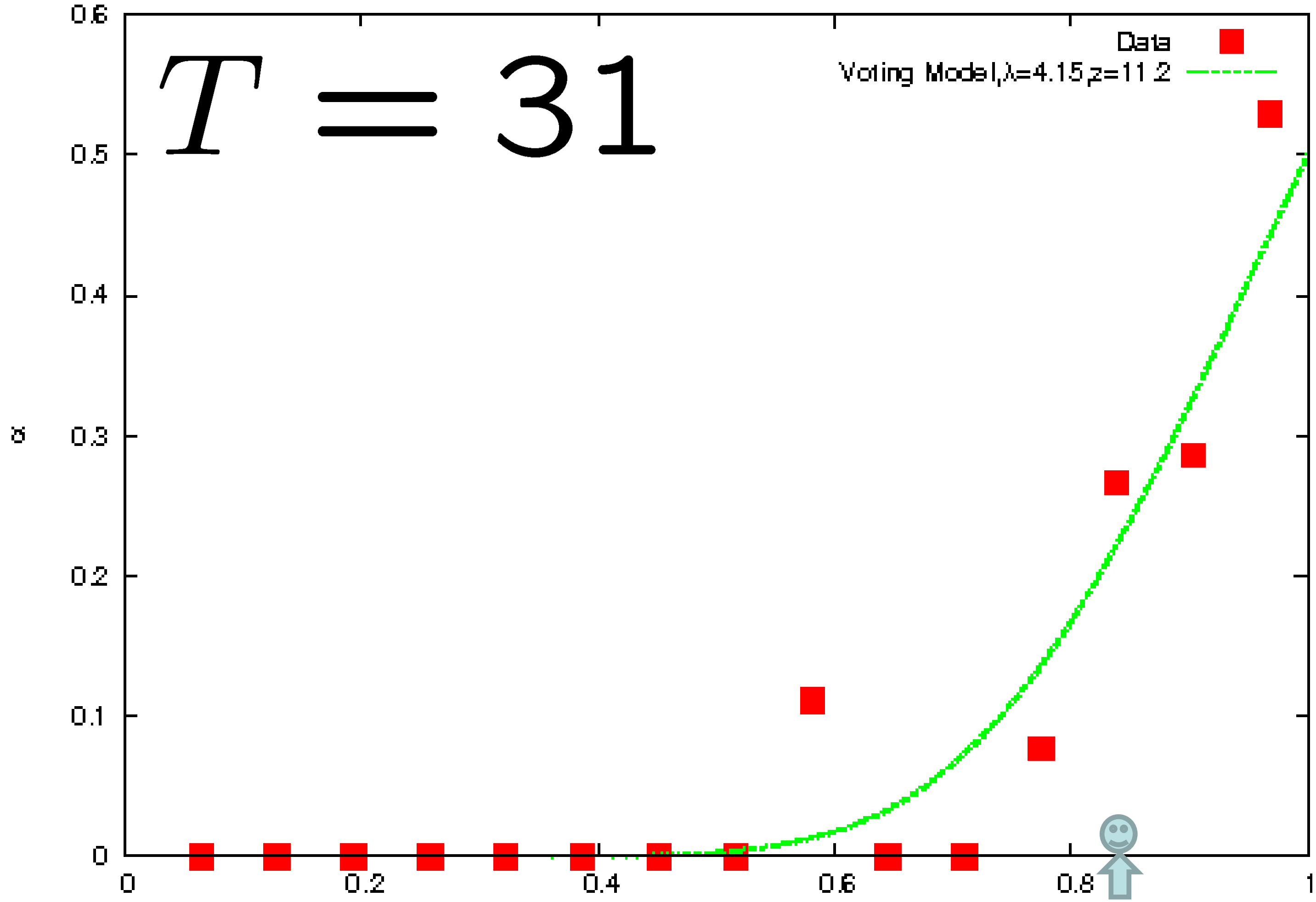


$\lim_{T \to \infty}$



α = Prob. **(All**  p% 

)

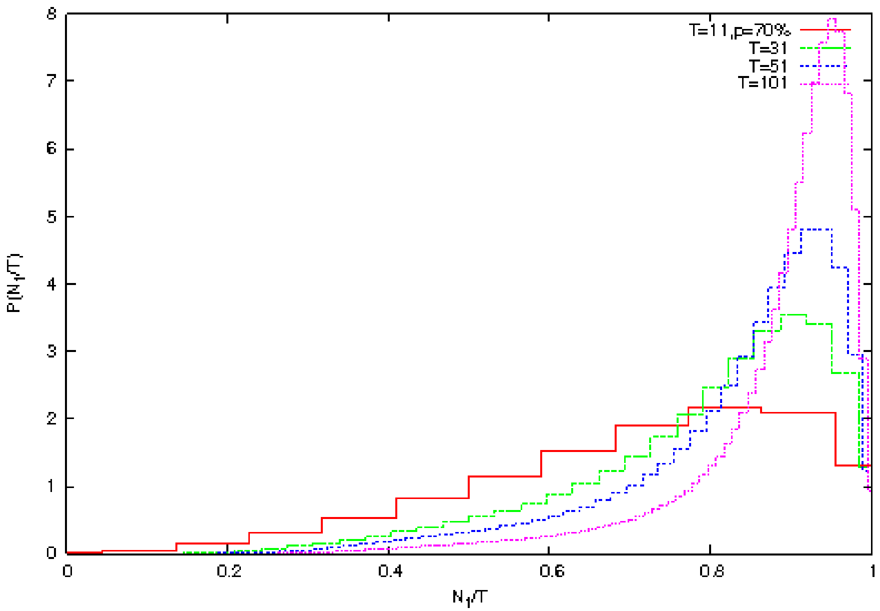


$$\lim_{T \to \infty}$$

How to detect p_c

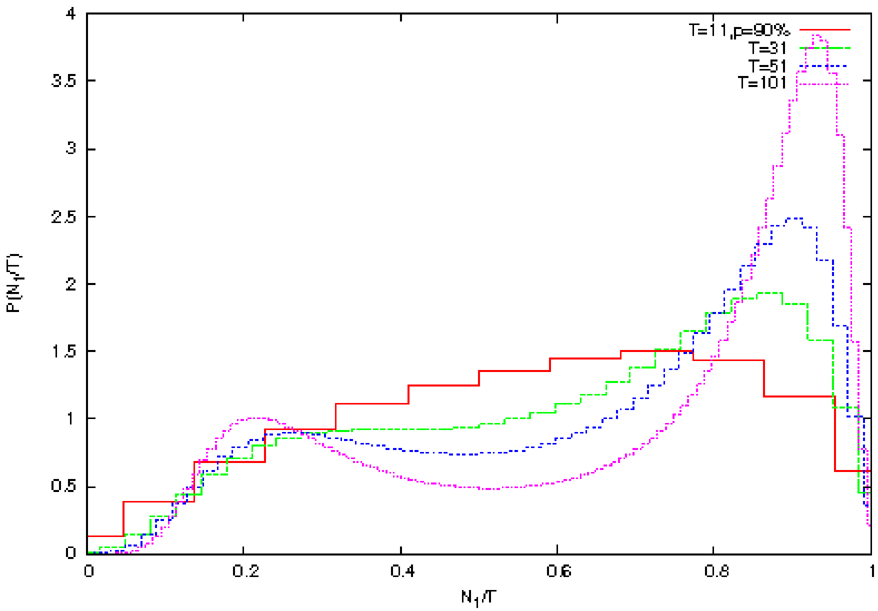
$$Z(t) = \frac{N_1^{\infty}(t)}{t}$$

$p < p_c$



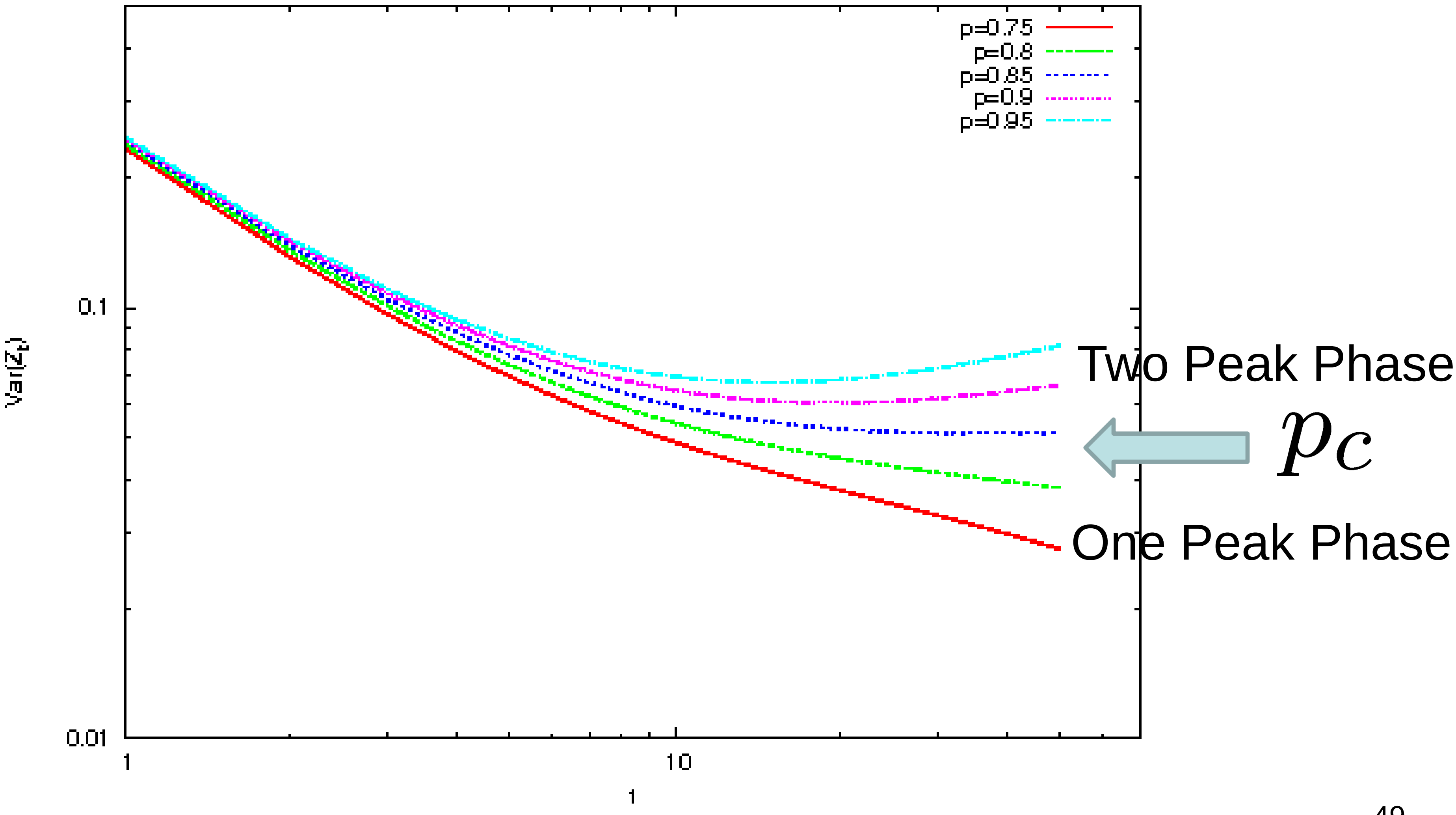
$$\lim_{T\to\infty} \text{Var}(Z(T)) \to 0$$

$p > p_c$

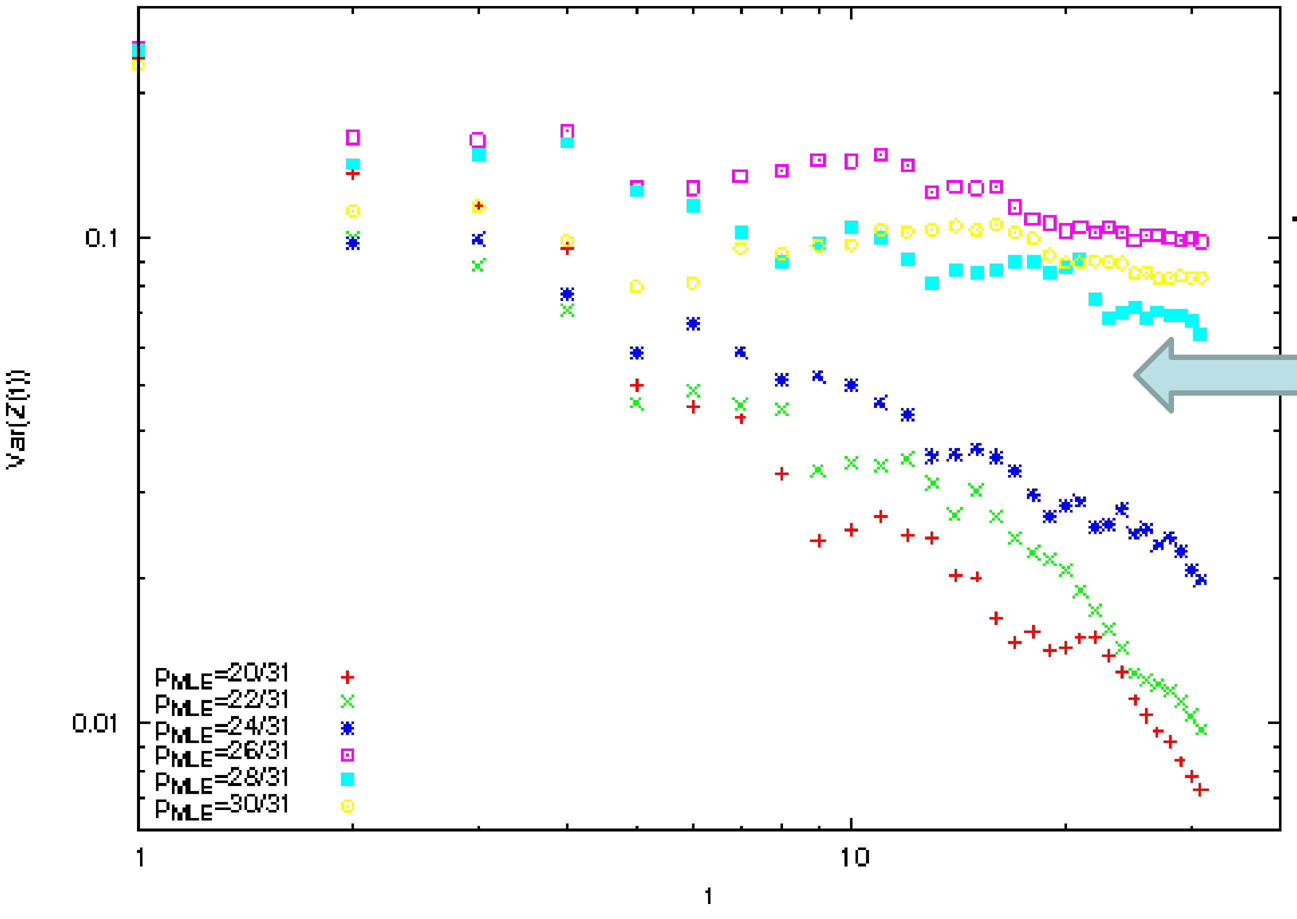


$$\lim_{T\to\infty} \text{Var}(Z(T)) \to c > 0$$

How to estimate p_c?



Phase Transition



Two Peak Phase

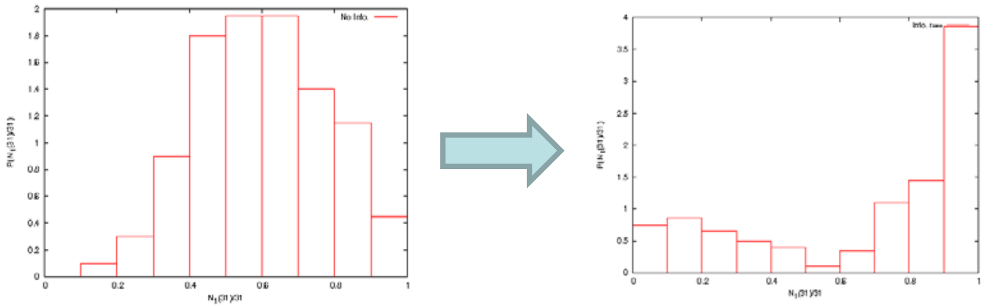
p_c

One Peak Phase

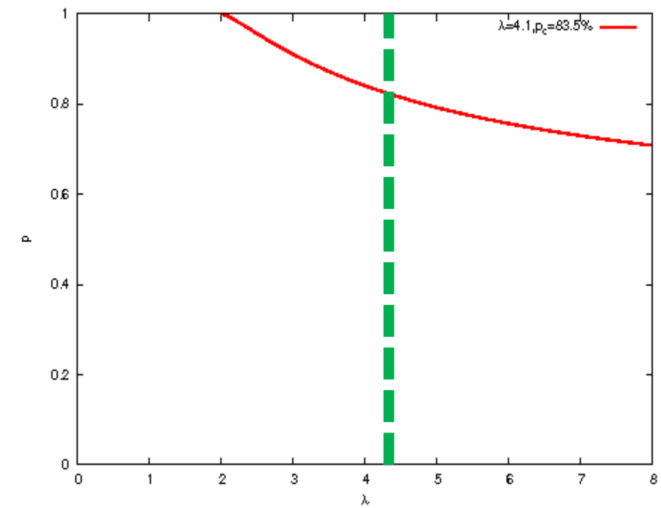
$$p_c = \frac{26}{31} \simeq 0.84$$

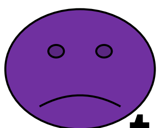
Conclusions 2

The infomation cascade is a phase transition.



Phase Diagram





How he votes ?

t+1

$$p_1(r, N_1^r) = \frac{1}{2}(\tanh \lambda \left(\frac{N_1^r - \frac{1}{2}r}{r+z} \right) + 1)$$

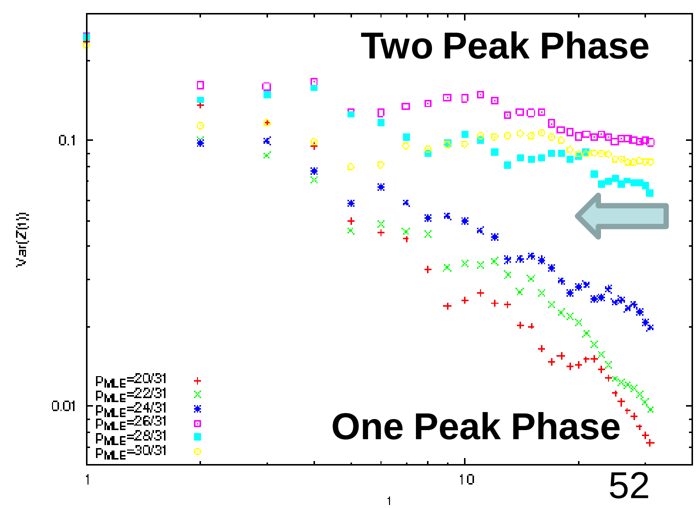
$$\lambda_{MLE} = 4.15, \quad 3.57 \leq \lambda \leq 4.92 (95\% \text{Conf.})$$

$$z_{MLE} = 11.21, \quad 8.72 \leq z \leq 14.64 (95\% \text{Conf.})$$

$$p_c = 0.835 \simeq \frac{26}{31}$$

Mean Field

Exp.



$$p_c \simeq 80\%$$

$$p > p_c$$

$$\Pr(\text{Majority=False}) = \alpha > 0$$

Thank you !