2011.09.07@JWEIN11

情報カスケードのミクロとマクロ

投票実験と統計物理による解明

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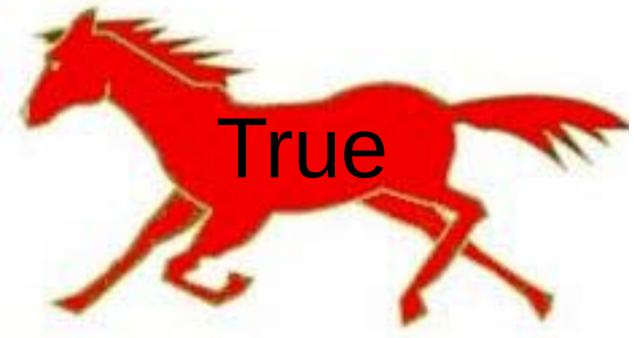
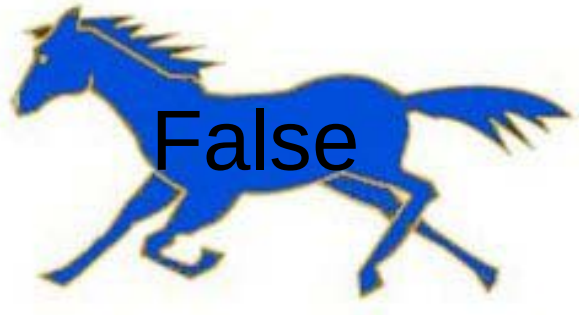
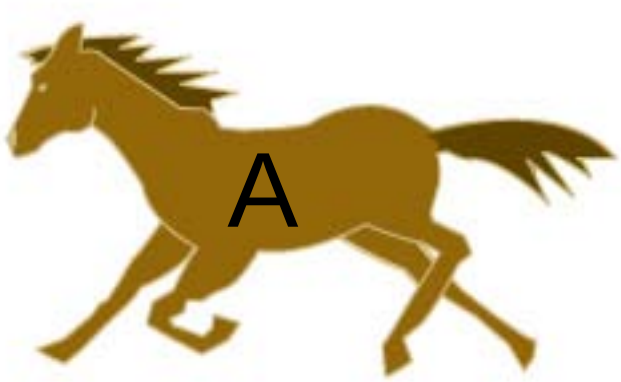
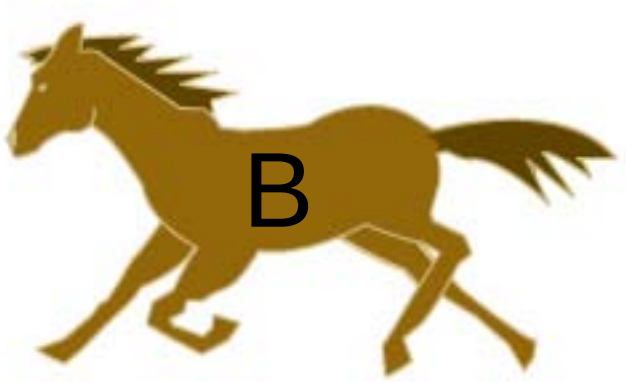
久門　正人　　Standard and Poors
高橋　泰城　　北海道大学・文学部

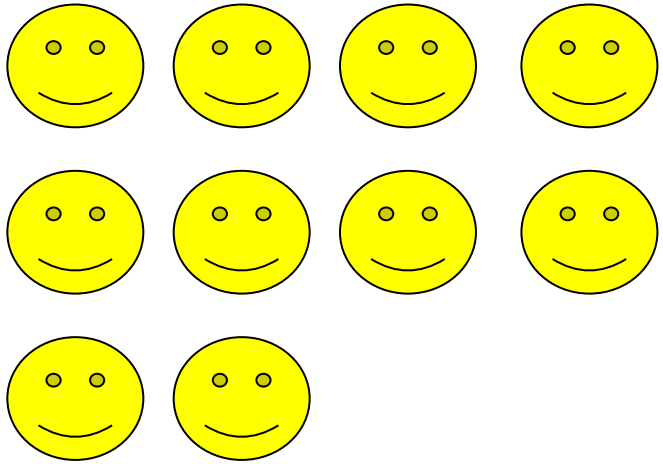
Quiz : A or B









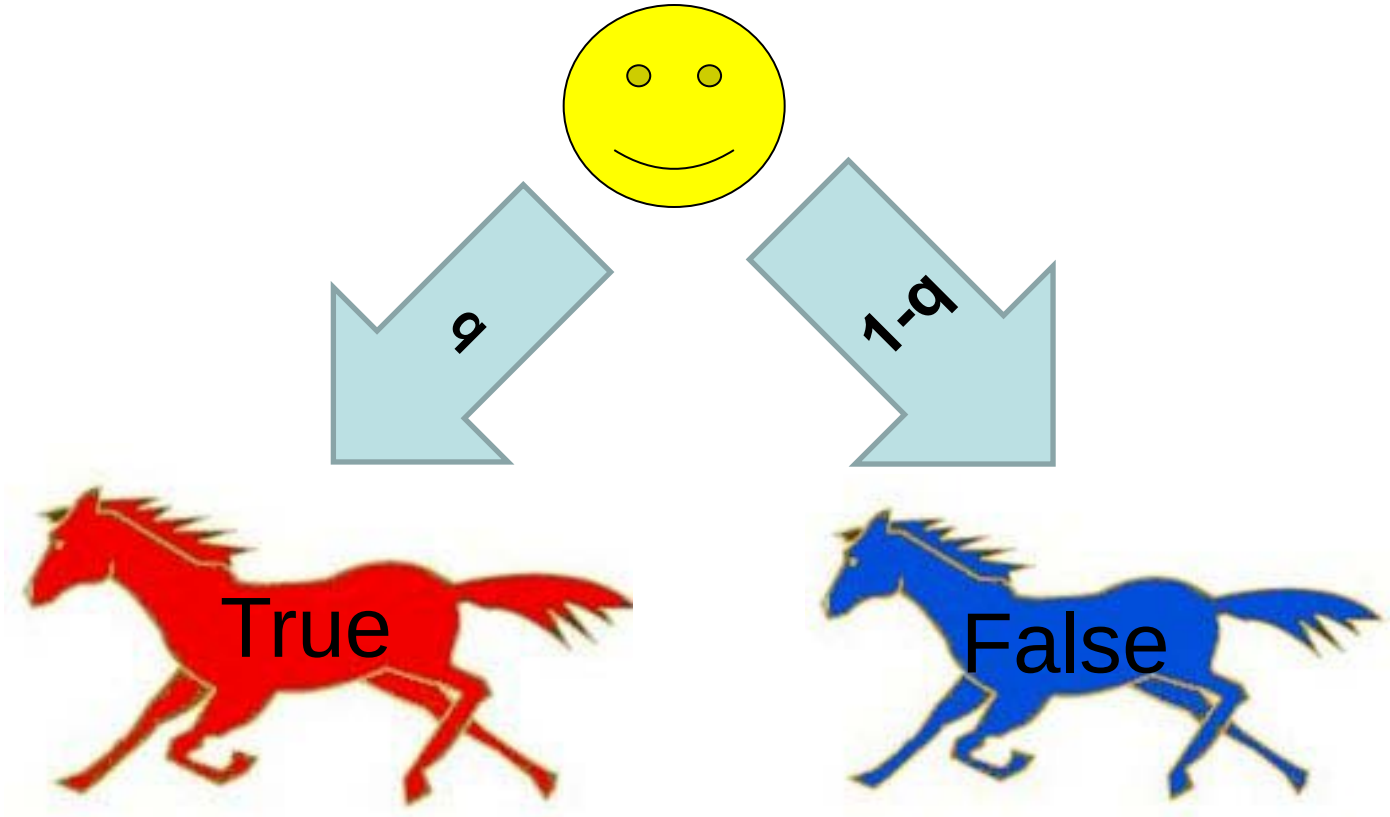


Can you trust others' choices ?

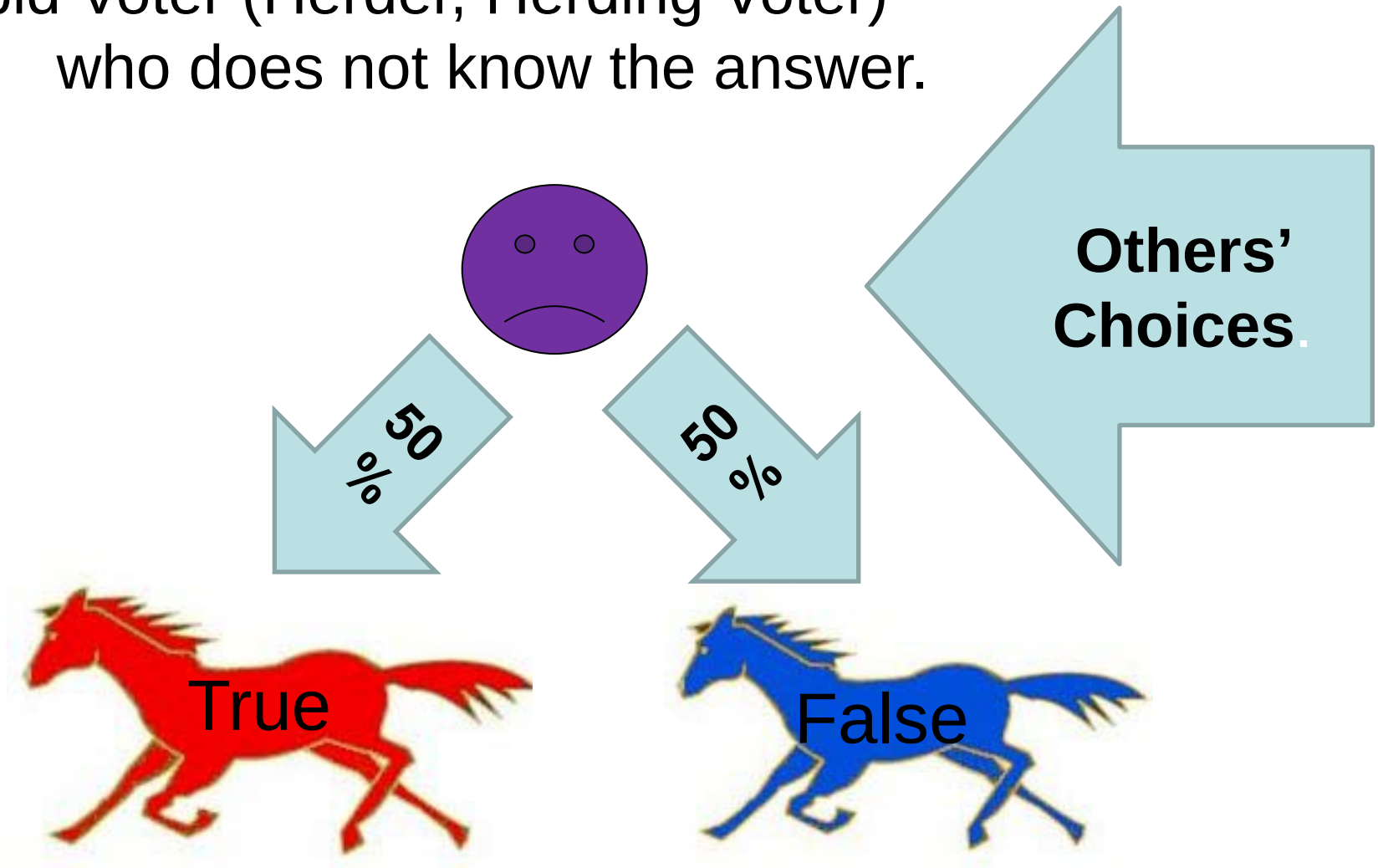
Independent Voter

who knows the answer with prob. q=100%.

who is not affected by others' choices.

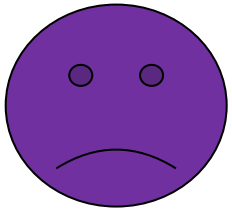


Stupid Voter (Herder, Herding Voter)
who does not know the answer.



Microscopic

Q.1



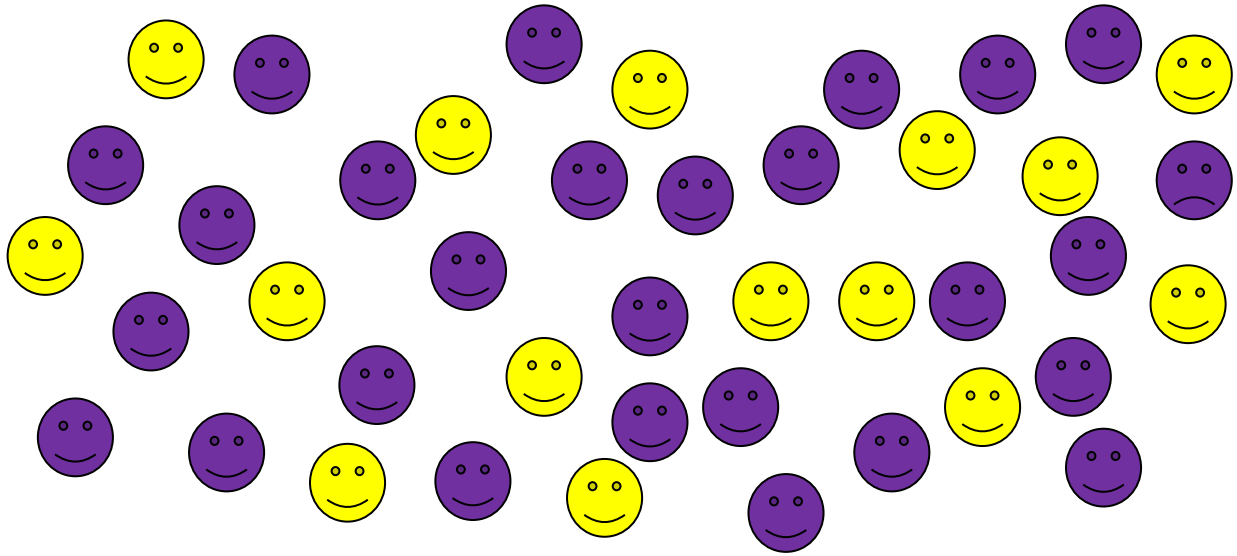
How he votes ?

Others' Choices

Macroscopic

Q.2

What happens ? $T \to \infty$

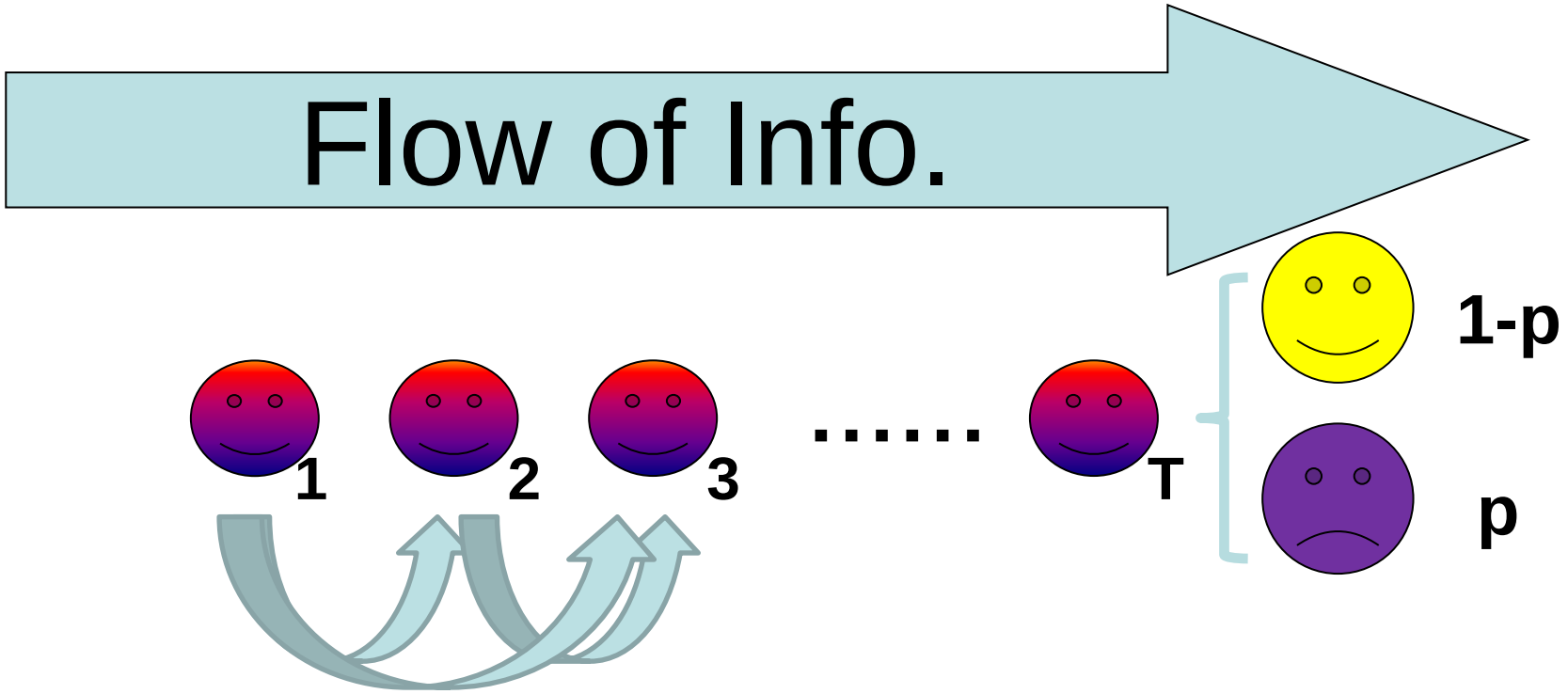


Q.3

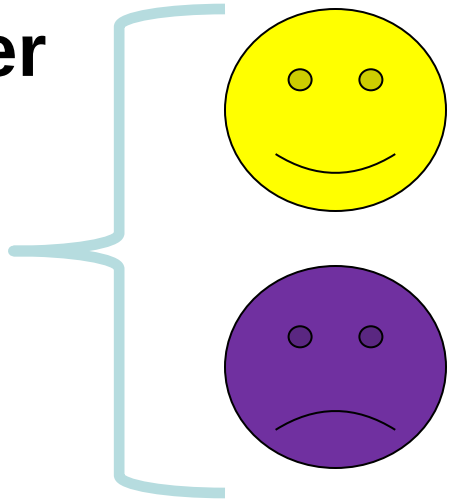
What is the Physics ?

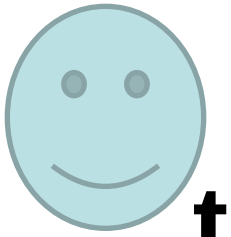
Theoretical Results

choose sequentially.



Voting Model with General Herder





t+1

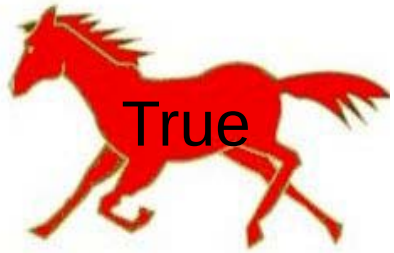
$(1-p)+p\cdot p_1(t,N_1^{\infty}(t))$ $\quad$ $p\cdot p_1(t,N_0^{\infty}(t))$

$$p_1(t,N_1^{\infty})=\frac{1}{2}\left(\tanh\lambda\left(\frac{N_1^{\infty}-\frac{1}{2}t}{t+z}\right)+1\right)$$

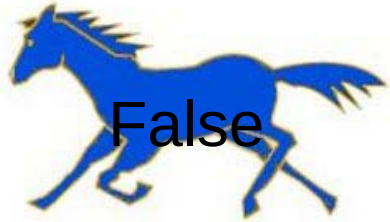
Truth $N_1^{\infty}(t)$

False $N_0^{\infty}(t)$

$$\lambda = \infty$$

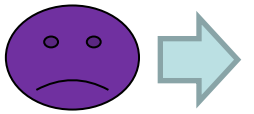
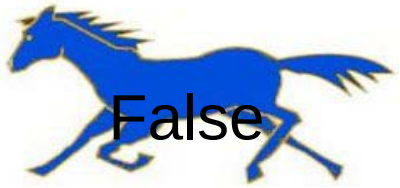


$$\hat{Z}(t) \equiv \frac{\hat{N}_1^{\infty}(t)}{t} \sim \alpha\delta_{1-p} + (1-\alpha)\delta_1 \quad t \to \infty$$



$$\alpha = \frac{2p - 1 + |2p-1|}{3 + |2p-1|}$$

Digital herders and phase transition in a voting model
M.Hisakado and S.Mori,J.Phys.A,Math.Theor.44(2011)275204

α = Prob. **(All**  

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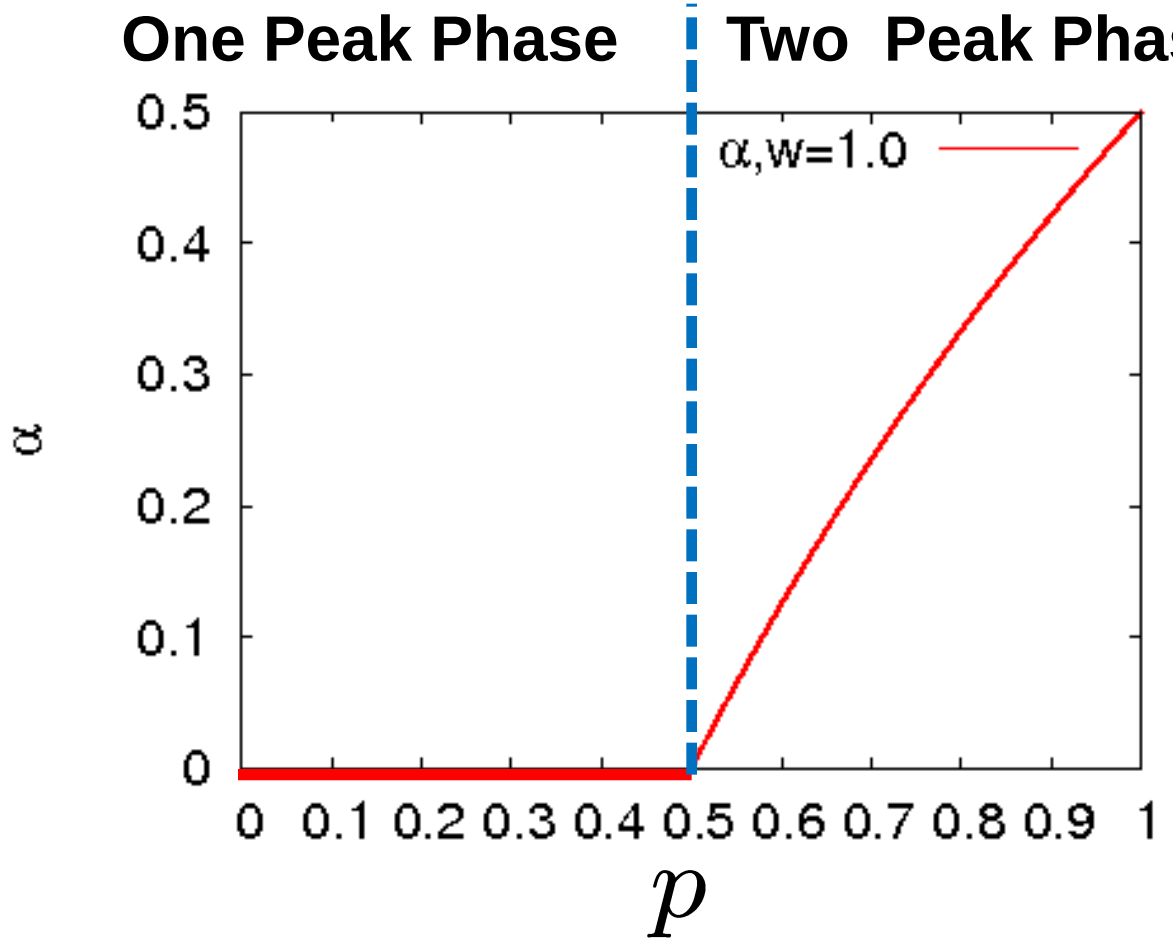
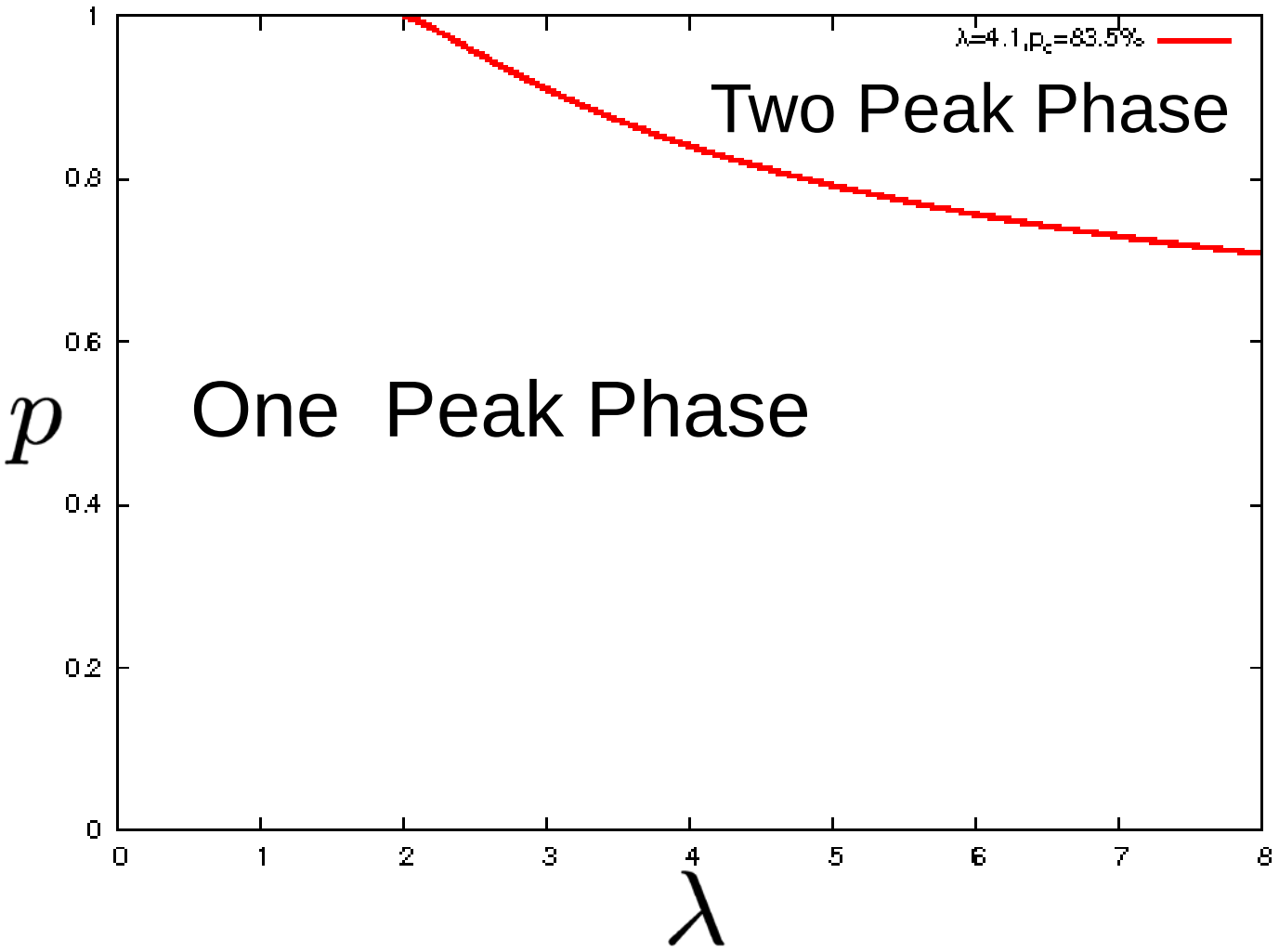


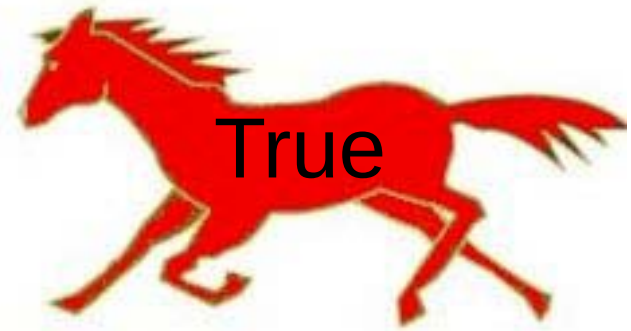
$$p_c = \frac{1}{2}$$

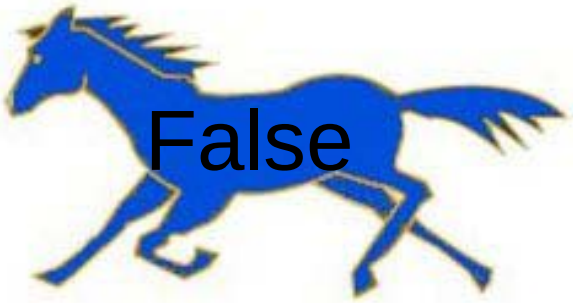
Phase Diagram



Experimental Results

Quiz : A or B





One by one

3000 yen + 1000yen for Top 10 students.
31(=T) *2=62 students,100 questions.

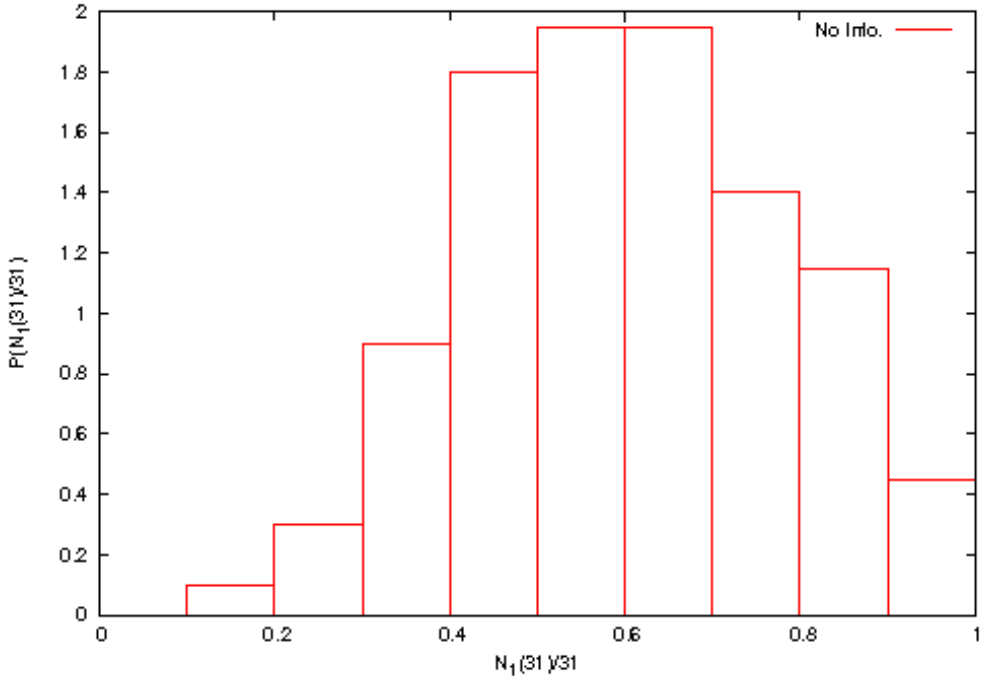
14: How much is the price on Buggy the Clown ?
The captain of Buggy's pirates.（at 2010.9.1)

A. 13 millions B
B. 15 millions B

$T = 31,200 \text{quiz}$

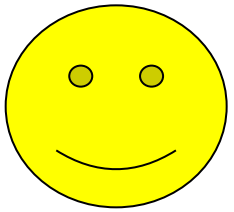
No Information Flow

$$P(N_1(T)/T)$$



$$< N_1(T)/T > \simeq 0.6$$

$$E\left[\frac{\hat{N}_1(T)}{T}\right] = (1-p) + \frac{1}{2}\cdot p \Rightarrow p \simeq 0.8$$



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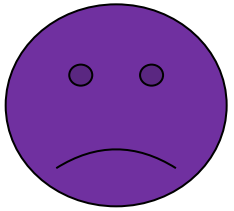
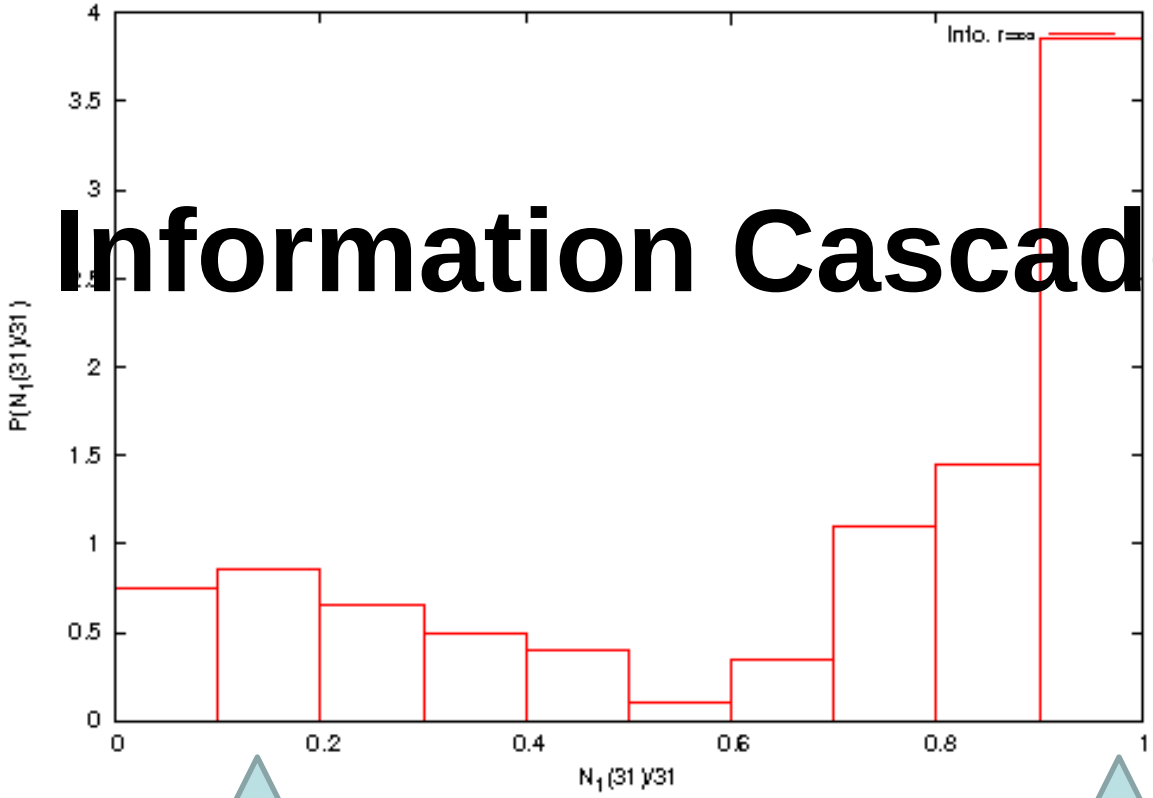


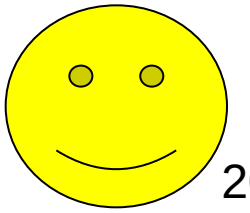
80%

Information Flow

$P(N_1^{\infty}(T)/T)$



Information Cascade



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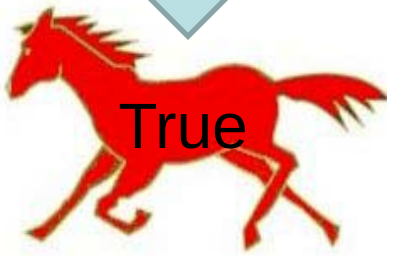

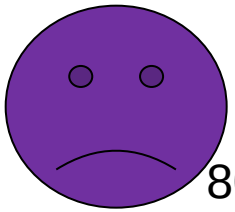

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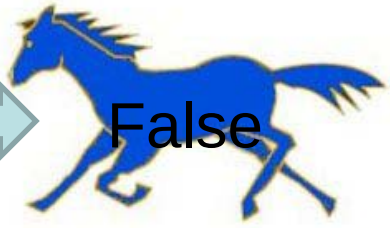

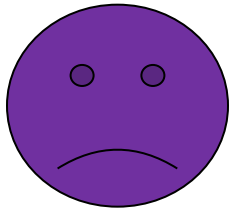

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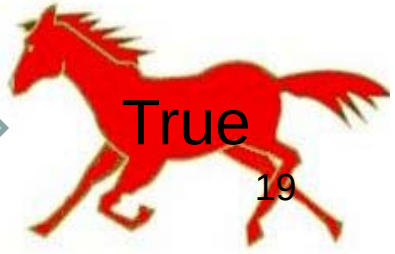
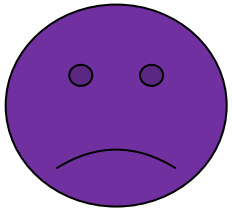


Q.1

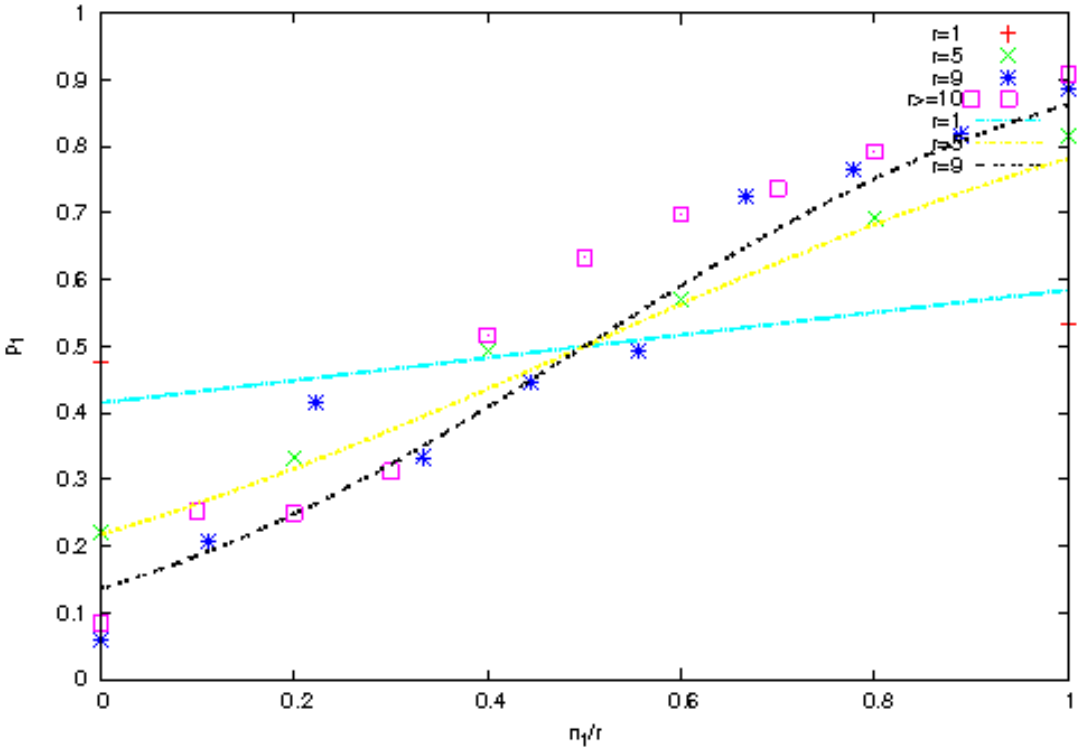


How he votes ?

$p_1(r, N_1^r)$

$$\left\langle \frac{\hat{X}^r_{t+1} - \hat{z}^0_T}{1 - \hat{z}^0_T} \middle| \sum_{s=1}^{t} \hat{X}^r_s = n_1 \right\rangle$$

$$\hat{z}^0_T \equiv \frac{1}{T} \sum_{t=1}^{T} \hat{X}^0_t$$



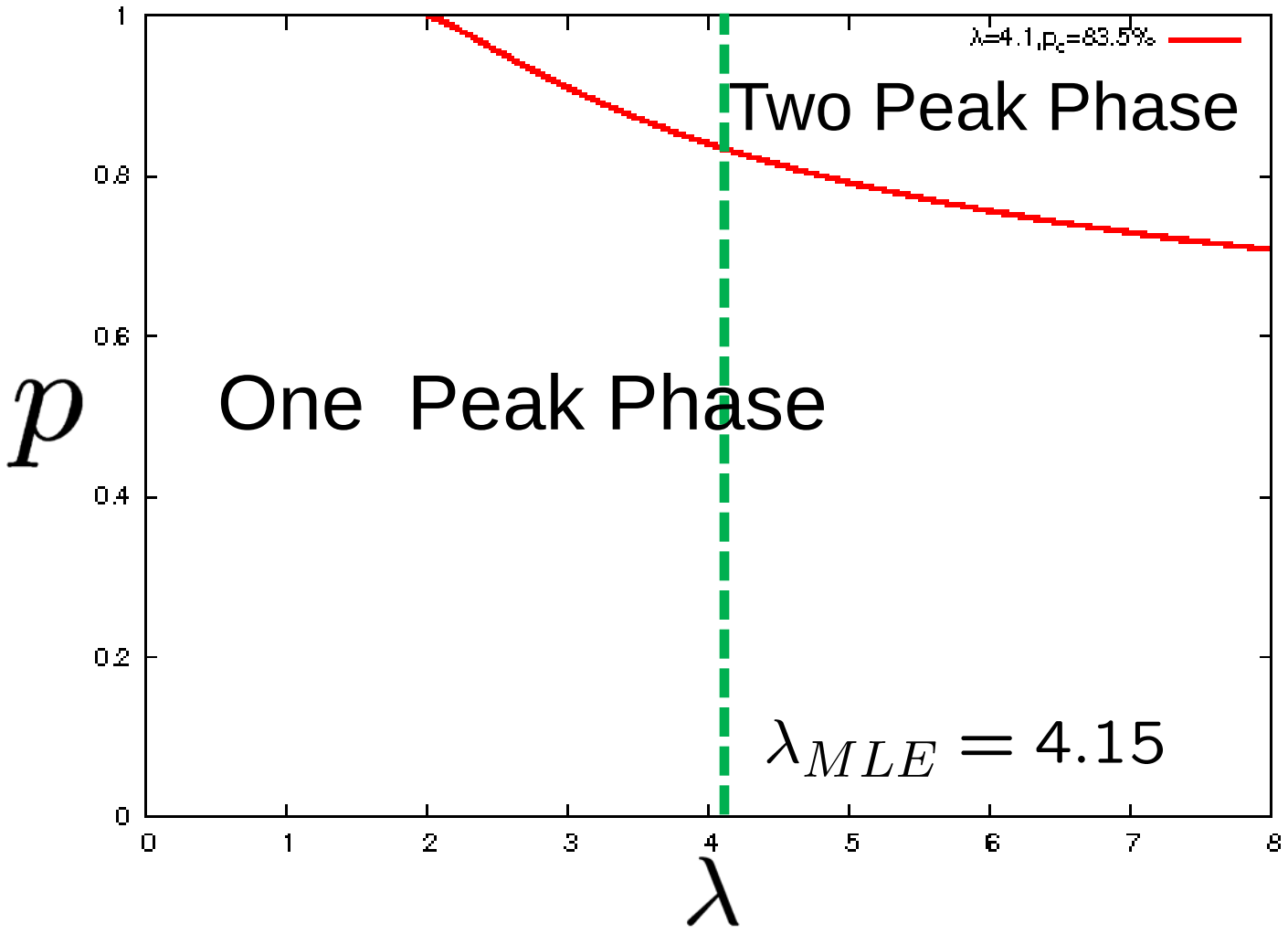
$$\frac{N_1^r}{r}$$

$$p_1(r, N_1^r) = \frac{1}{2}(\tanh \lambda \left(\frac{N_1^r - \frac{1}{2}r}{r + z} \right) + 1)$$

$\lambda_{MLE} = 4.15, \quad 3.57 \leq \lambda \leq 4.92 (95\% \text{Conf.})$

$z_{MLE} = 11.21, \quad 8.72 \leq z \leq 14.64 (95\% \text{Conf.})$

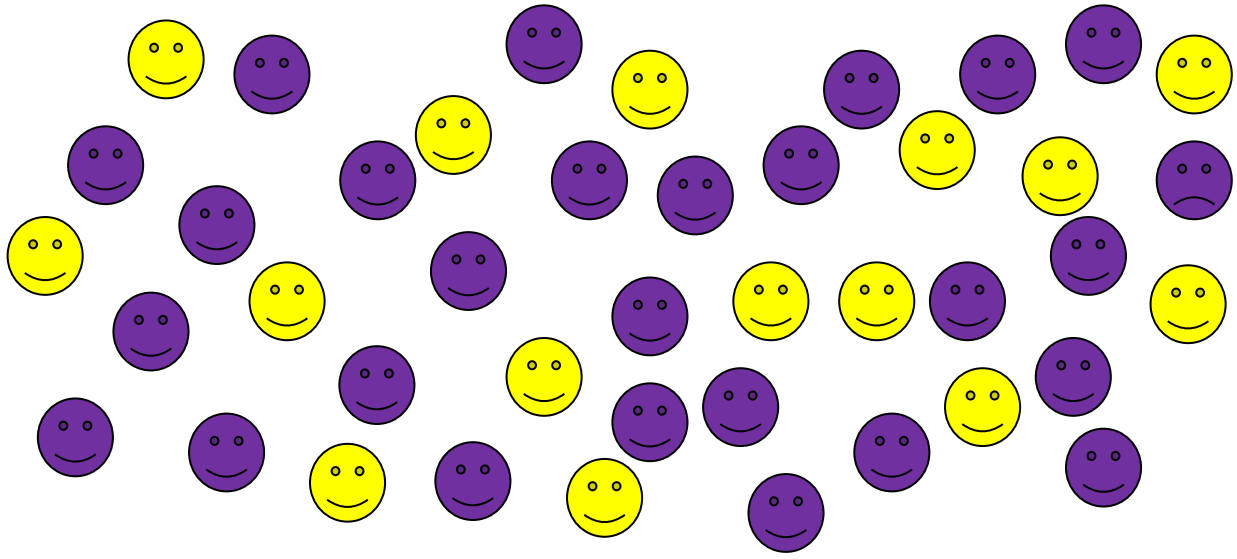
Phase Diagram

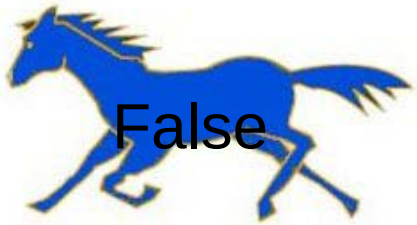


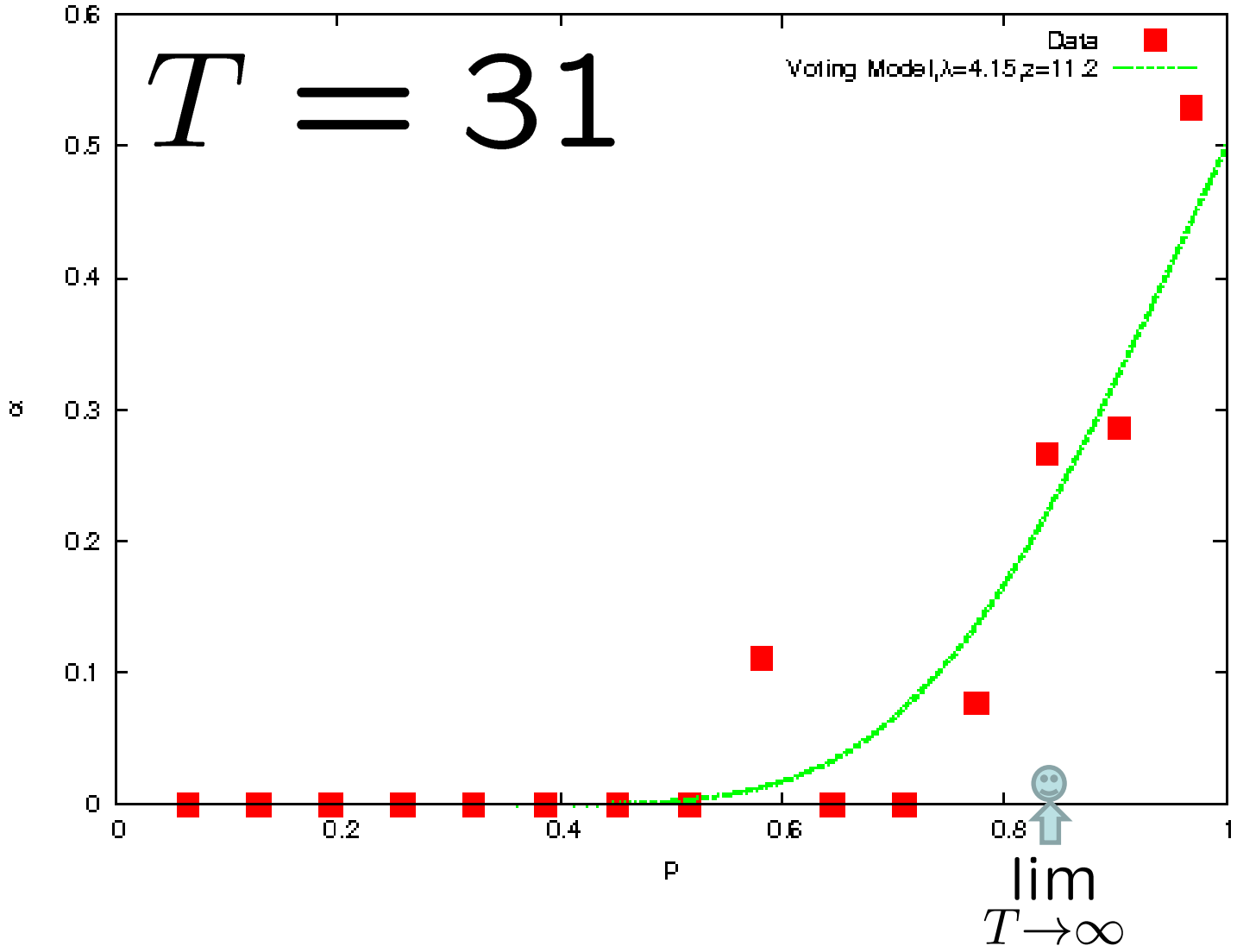
Macroscopic

Q.2

What happens ? $T \to \infty$



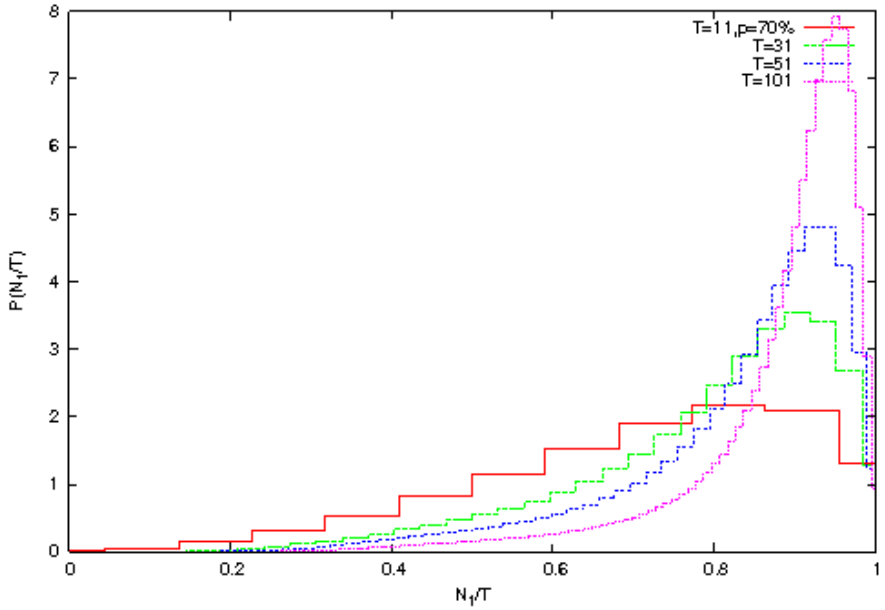
α = Prob. (**All**  p%  False)



How to detect p_c

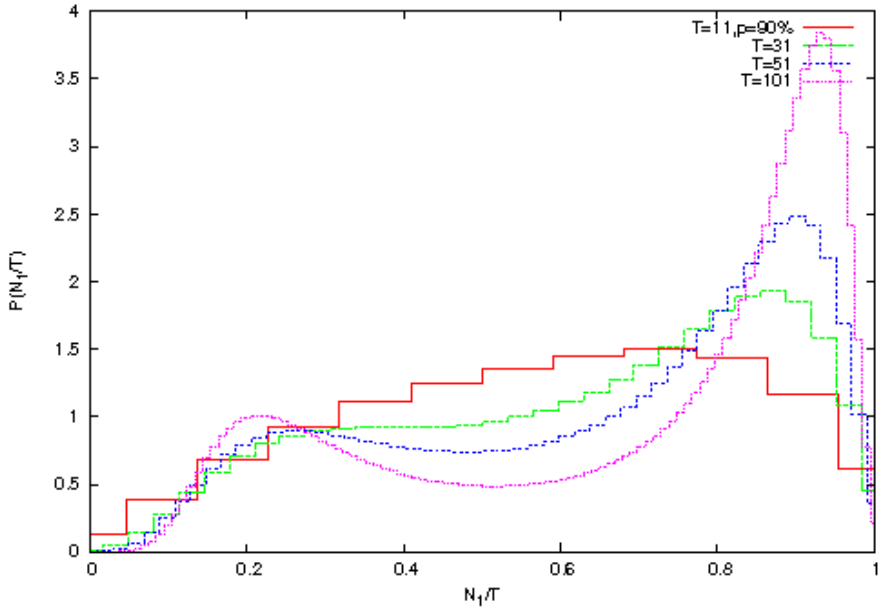
$$Z(t) = \frac{N_1^{\infty}(t)}{t}$$

$p < p_c$



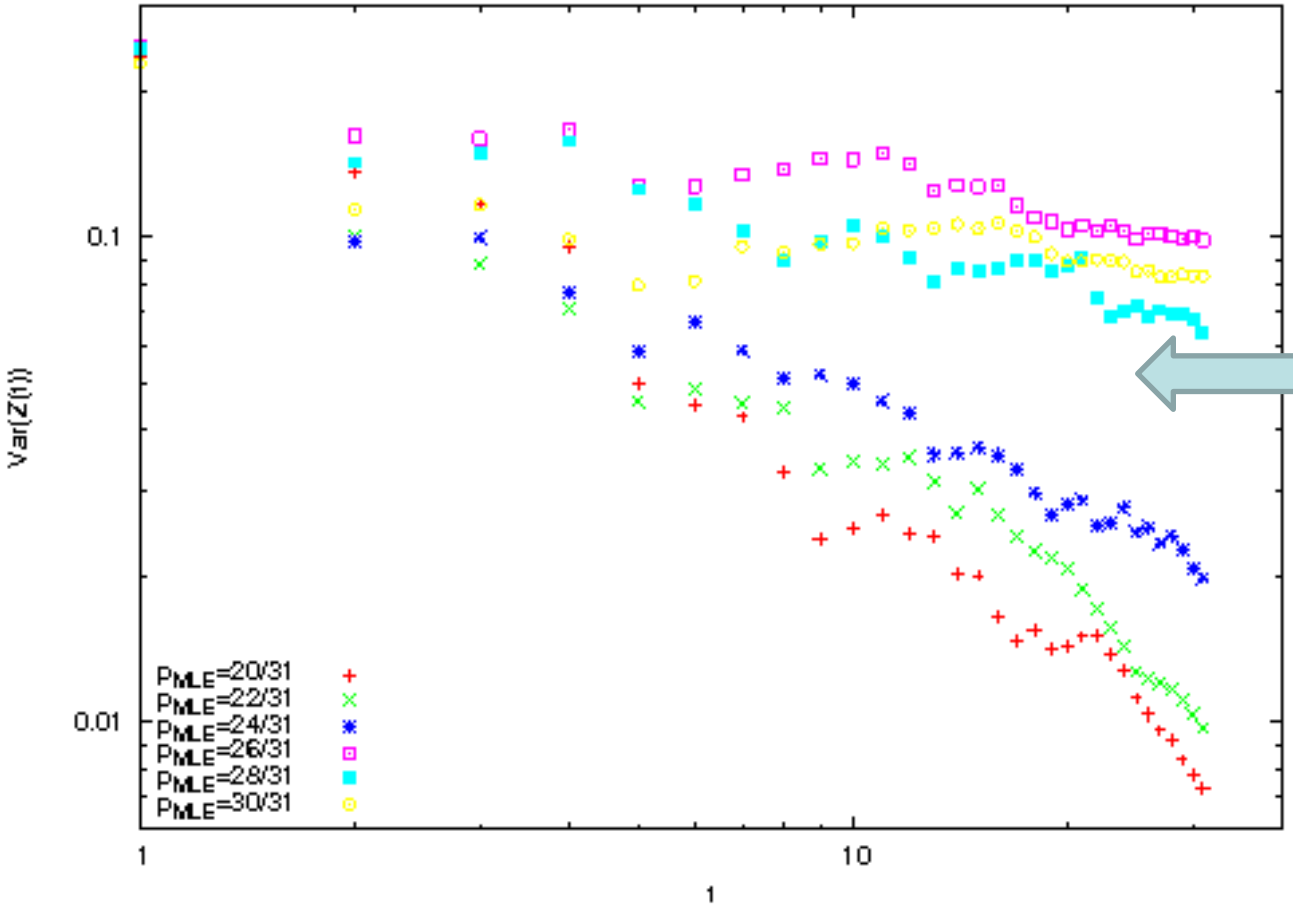
$\lim_{T\to\infty} \mathrm{Var}(Z(T)) \to 0$

$p > p_c$



$\lim_{T\to\infty} \mathrm{Var}(Z(T)) \to c > 0$

Phase Transition



Two Peak Phase

p_c

One Peak Phase

$$p_c = \frac{26}{31} \simeq 0.84$$

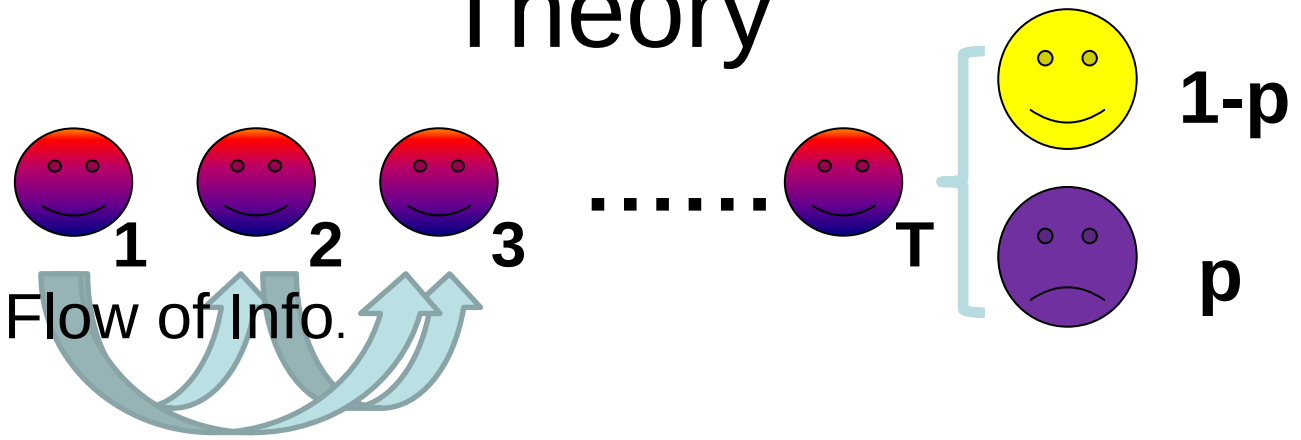
Q.3

What is the Physics ?

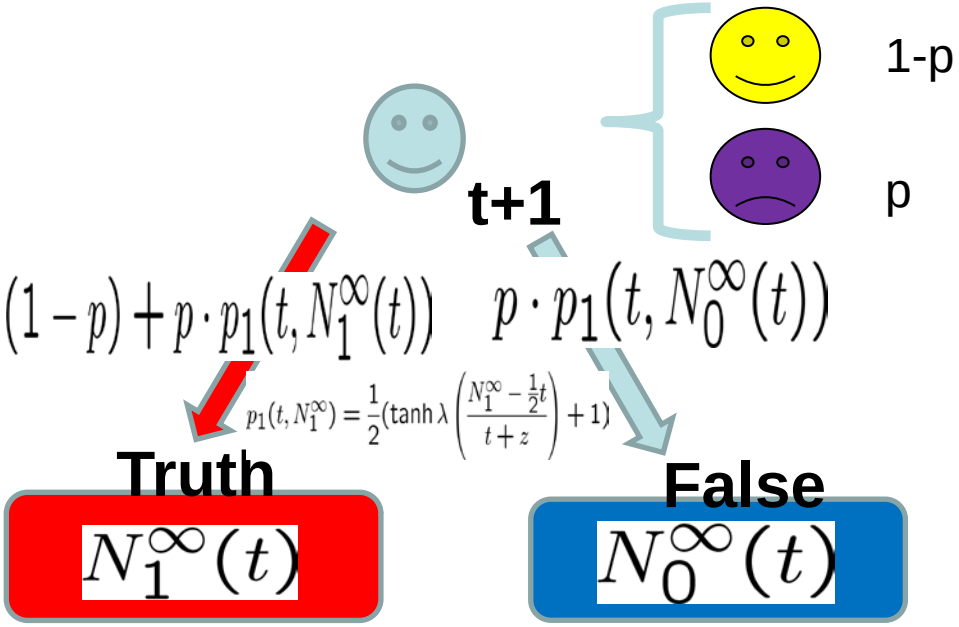
Phase Transition.

Conclusions

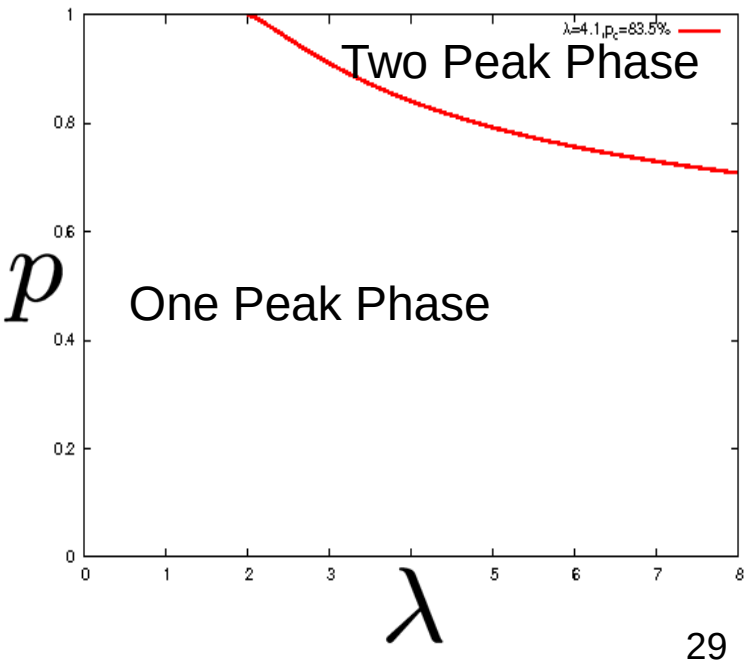
Theory



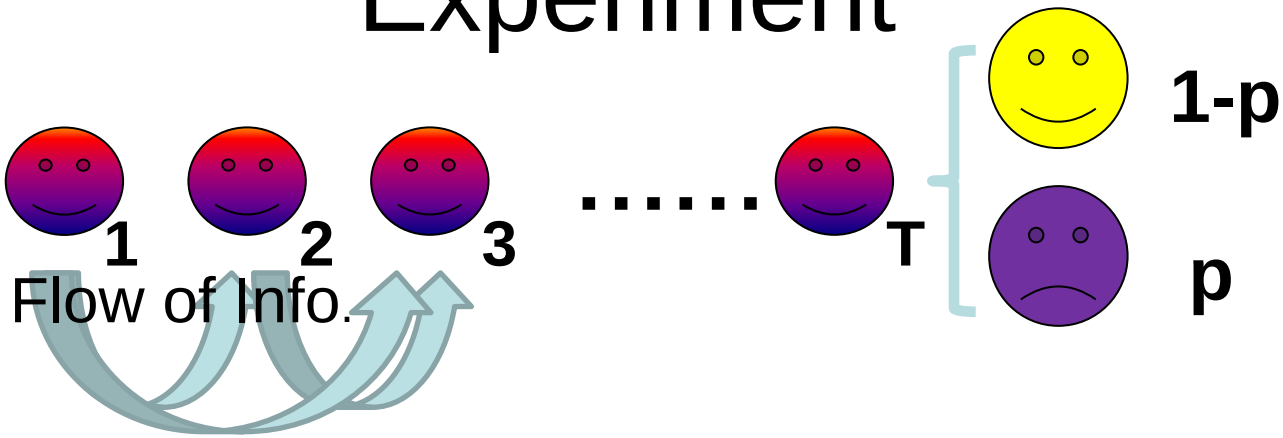
Voting Model with General Herder

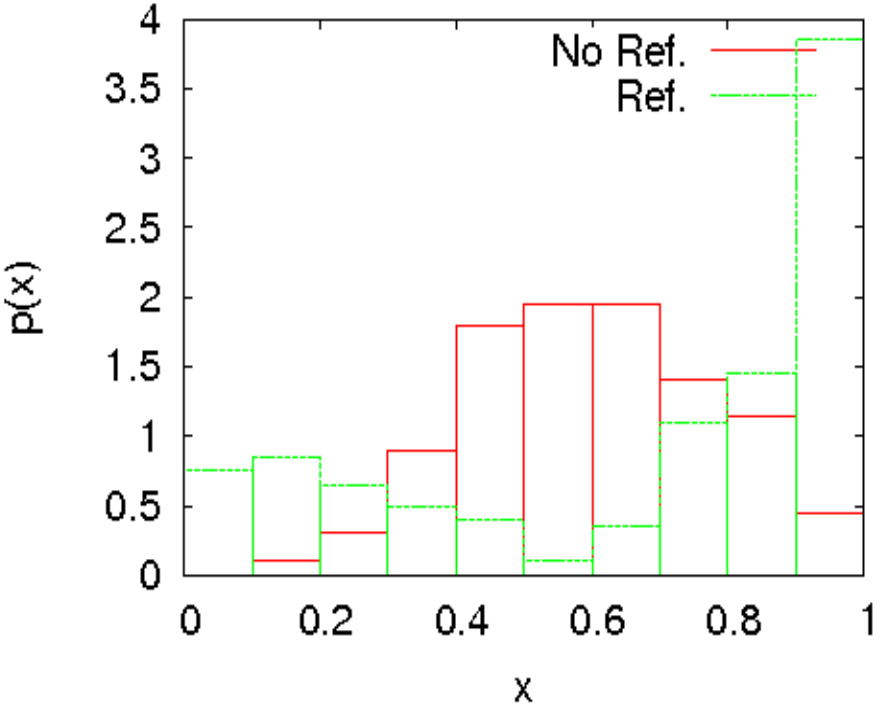


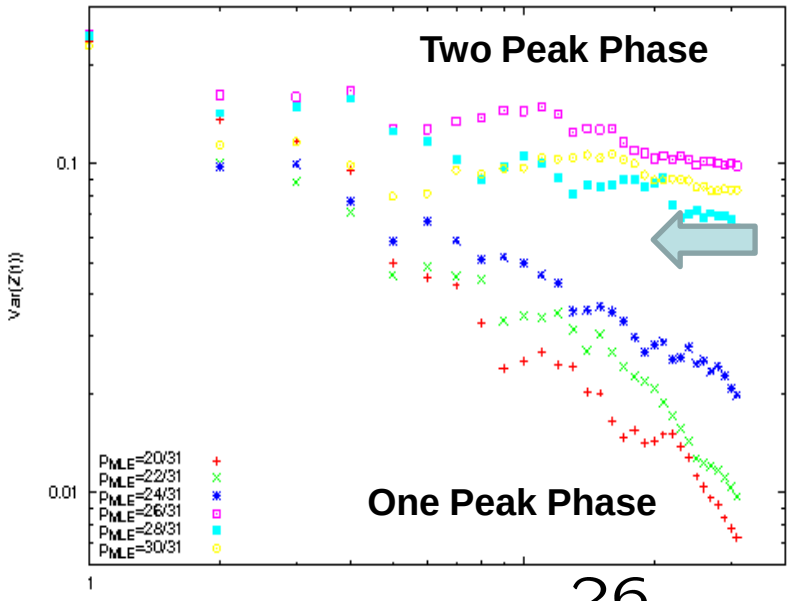
Phase Diagram



Experiment

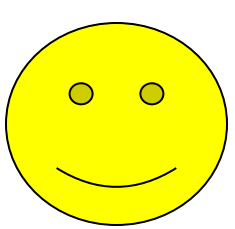




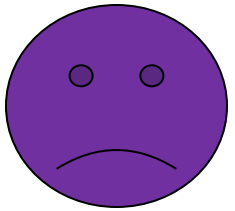


$$p_c = 0.835 \simeq \frac{26}{31}$$

Thank you ！



1-p

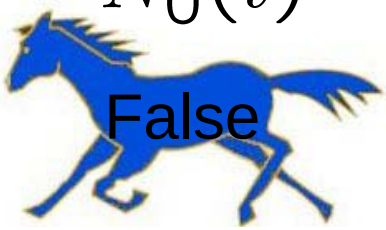


p

$$\hat{N}_1(t) + \hat{N}_0(t) = t$$

$\hat{N}_1(t)$

$\hat{N}_0(t)$



$$E\left[\frac{\hat{N}_1(t)}{t}\right] = (1-p) + 0.5 \cdot p = 1 - 0.5 \cdot p$$

$$E\left[\frac{\hat{N}_0(t)}{t}\right] = 0.5 \cdot p$$

A. 13 millions

B. 15 millions

Result

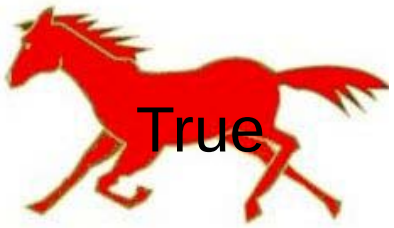
$$N_1(31) = 18$$
$$N_0(31) = 13$$

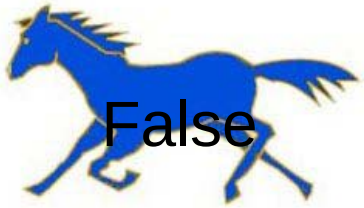
$$1 - \frac{1}{2}p = \frac{18}{31}$$

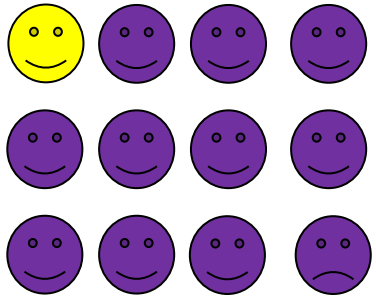
$$p_{\text{MLE}} = \frac{26}{31}$$

MLE=Most Likelihood Estimate

$1 - \alpha(\lambda, p)$

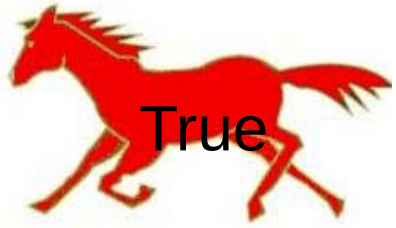


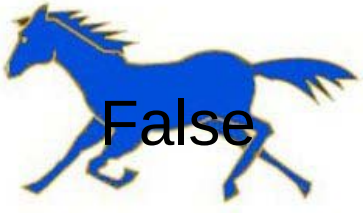


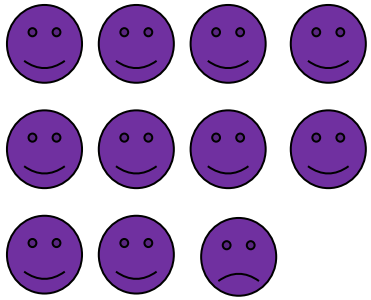


Majority is True !

$\alpha(\lambda, p)$







Majority is False !