平成24年度　統計数理研究所共同研究集会
「社会物理学の展望」

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場所：東京電機大学（千住キャンパス）二号館6階2602室

オッズのある二択の選択とMax-Min戦略

ヒトはオッズにどう反応するのか？

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問題：ロダンの「考える人」が肘をついている場所はどこ？

A：右脚
B：左脚

ヒント:人数

A：4人
B：6人

Return=1

ヒント:倍率

A：10/4 倍
B：10/6 倍

A：11/5 倍
B：11/7 倍

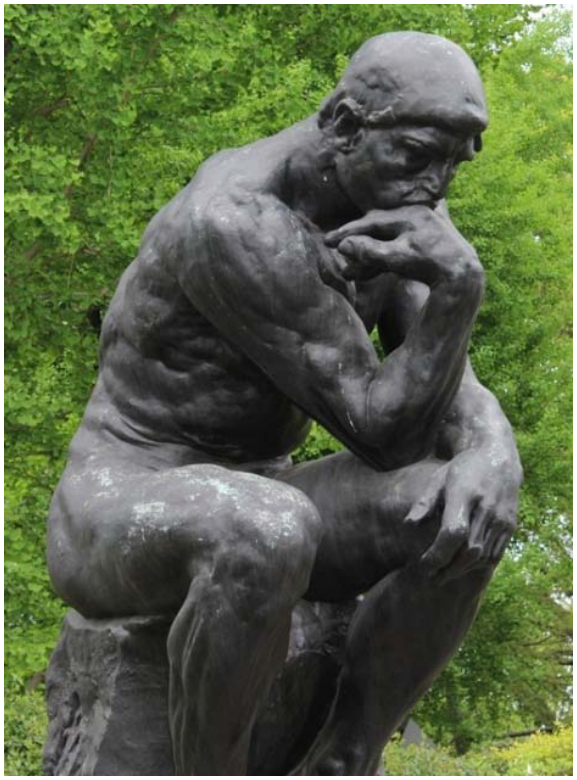
Return＝倍率

正解：B：左脚

ノーヒントでの正答率

グループA　53%
$T = 57$

グループB　46%
$T = 63$



ヒント:人数での正答率

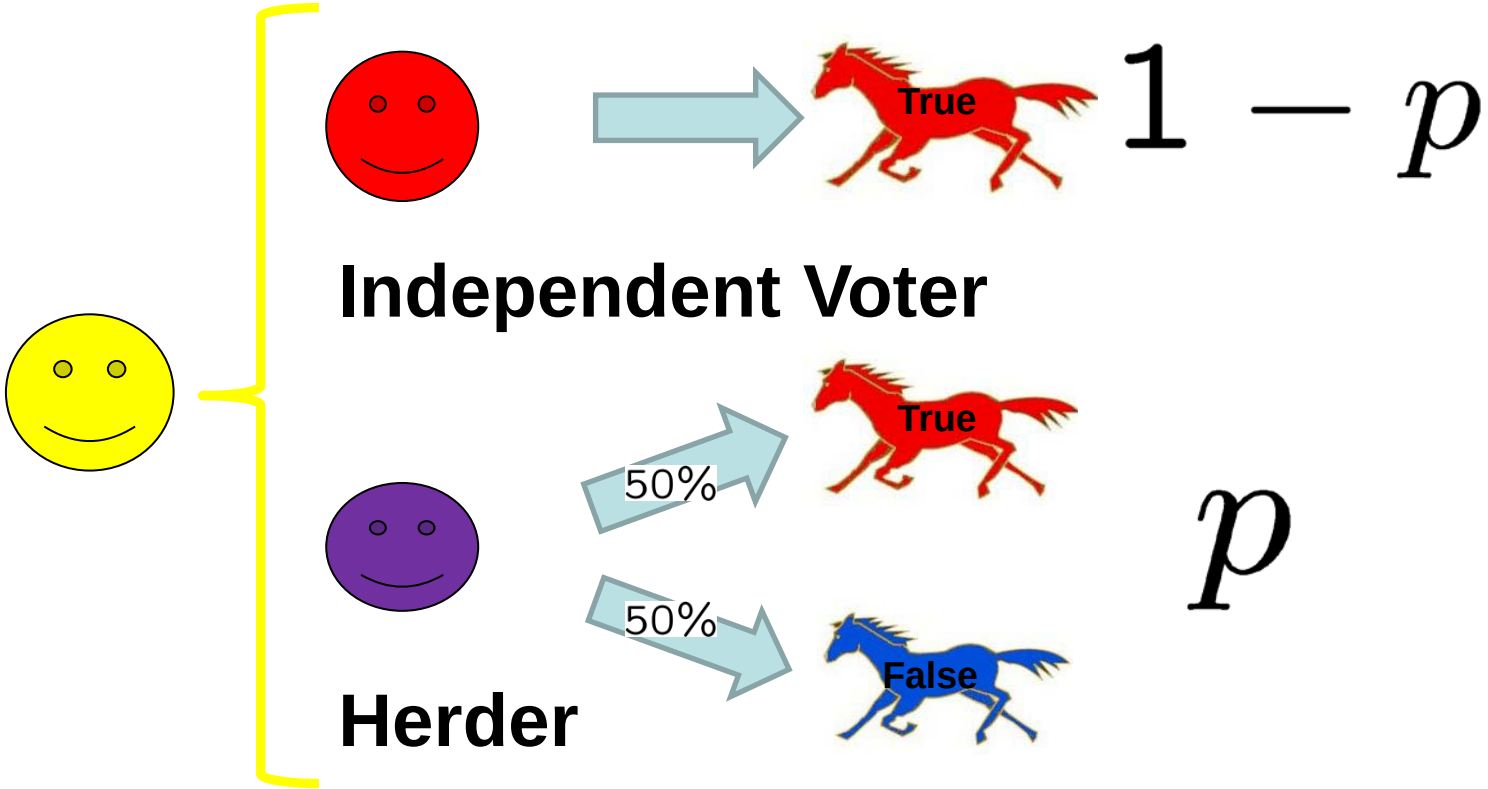
グループA　86%

グループB　16%

ヒント:倍率での正答率

グループA　74%

グループB　40%



平均正答率

$$q = (1-p)\cdot 1 + p\cdot\frac{1}{2} = 1 - \frac{1}{2}p$$

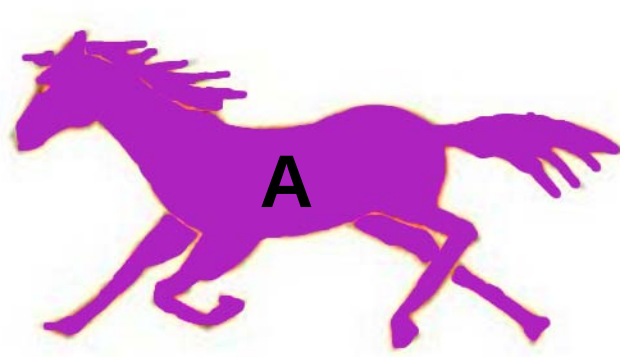
ヒント:人数

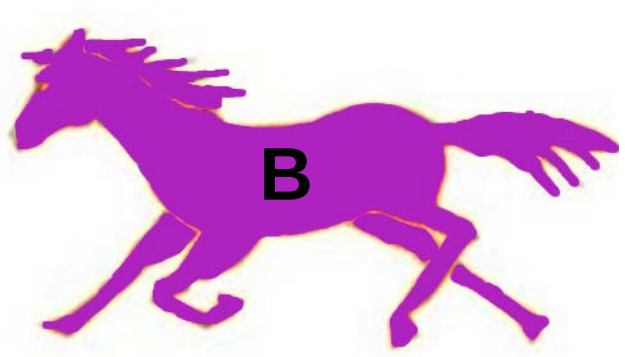
どう選択するのか？

t 人

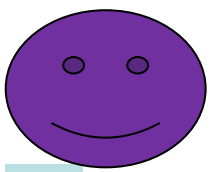
$$\{C_A(t), C_B(t)\}$$

?

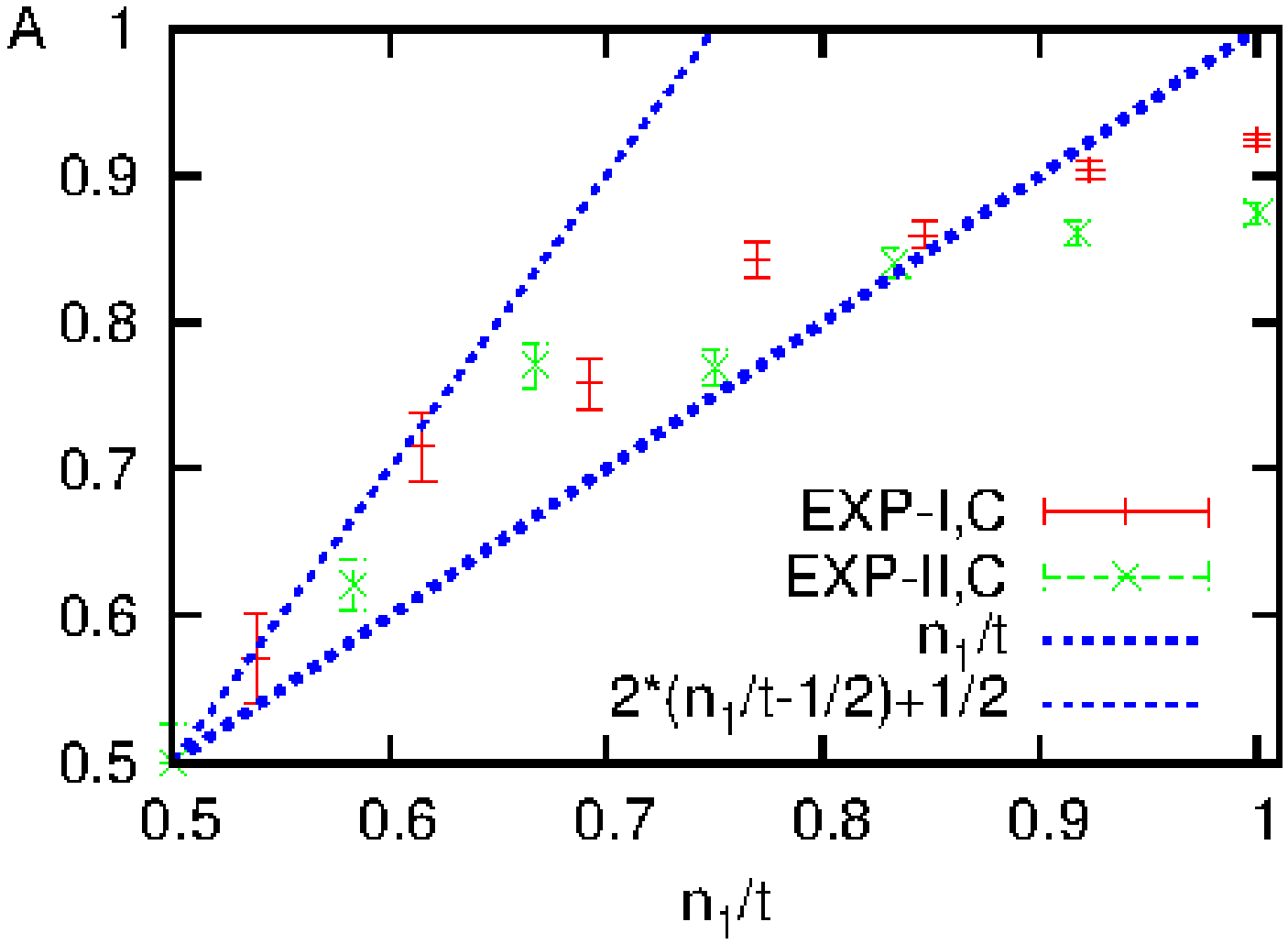




$q_h(t, n_1)$: Herder's Response Function



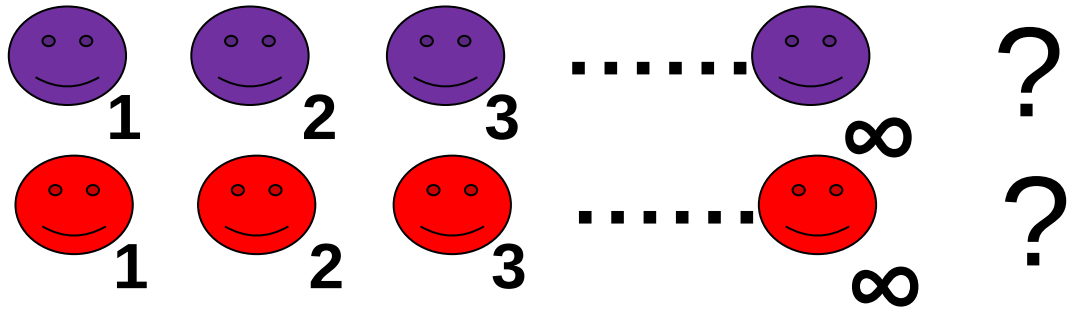
その選択肢を選ぶ確率

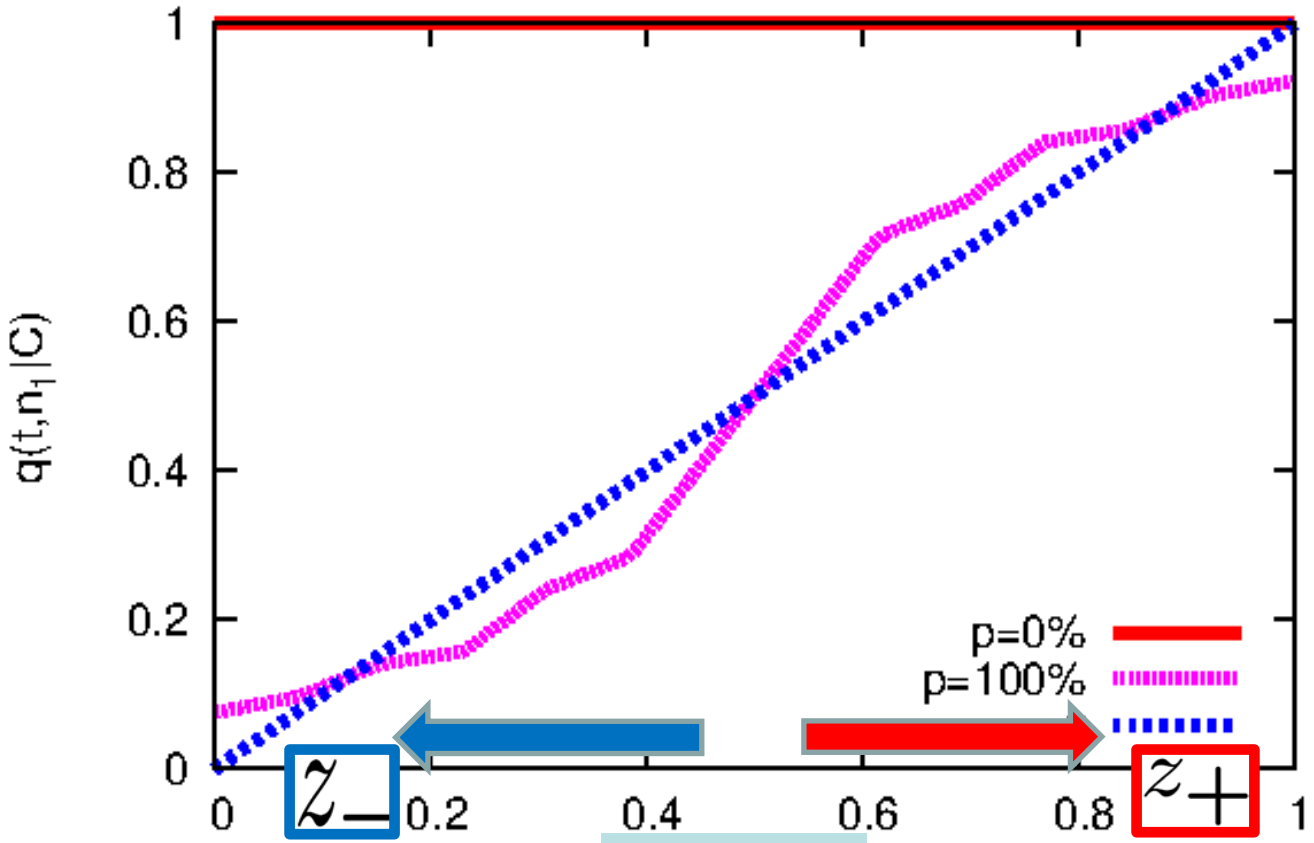


ある選択肢を選んだ人の比率

$p = 100\%$

$p = 0\%$





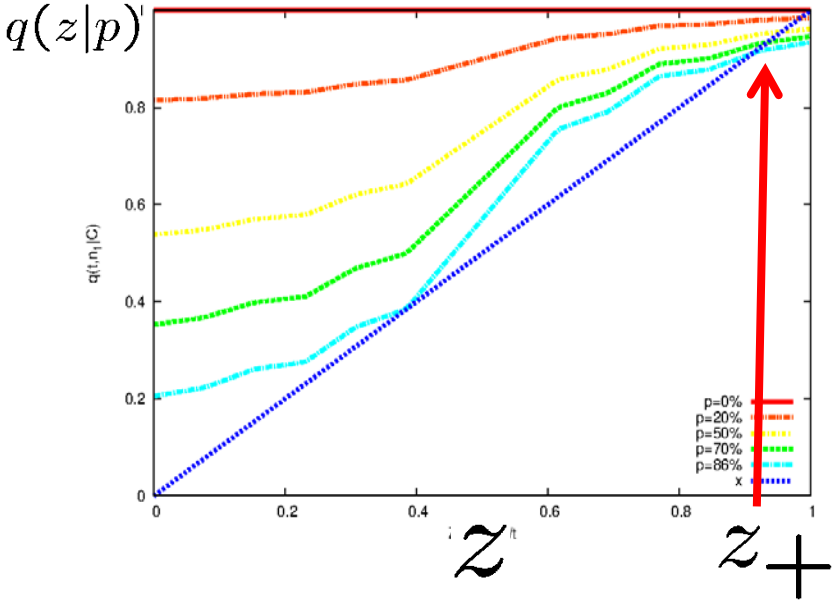
正答率

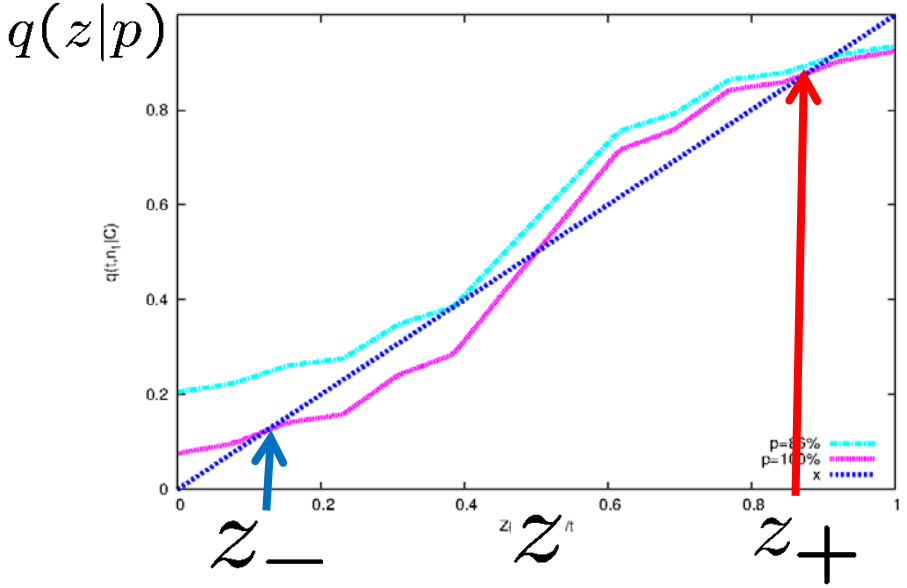
Self-Consistent equation

$$z = (1-p)\cdot 1 + p \cdot q_h(t, t\cdot z) = q(z|p)$$

$p \leq p_c = 86\%$

$p \geq p_c = 86\%$





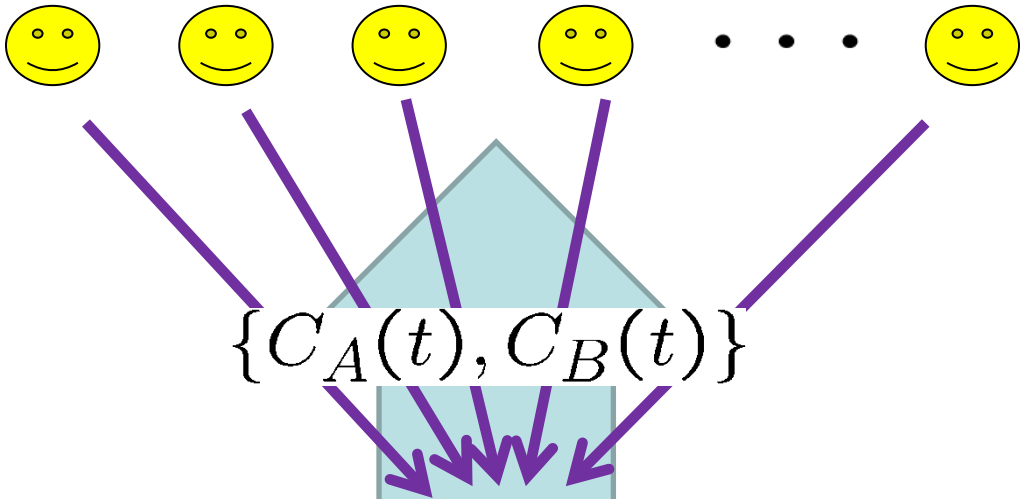
One-Peak Phase

Two-Peak Phase

Information Cascade Phase transition

S.Mori, M. Hisakado and T. Takahashi, PRE(2012).

ヒント:倍率

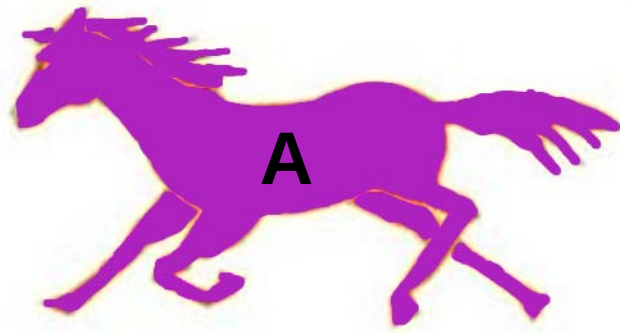


$\{C_A(t), C_B(t)\}$

Zero-sum game

$$M_A = \frac{t+1}{C_A(t)+1}$$

$$M_B = \frac{t+1}{C_B(t)+1}$$



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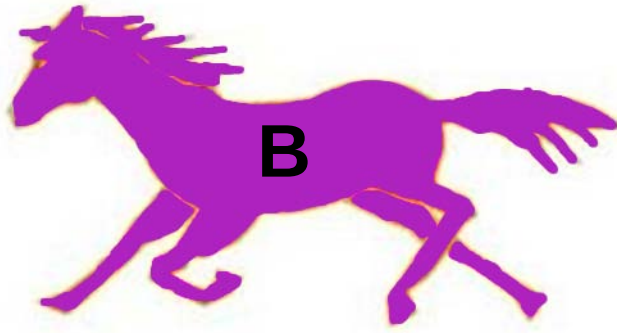
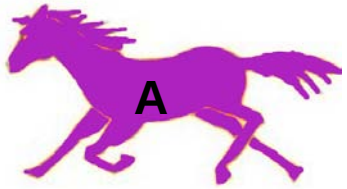


Zero-sum game

C_A

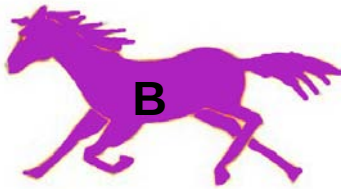
$M_A \cdot C_A$



$C_A + C_B + 1$

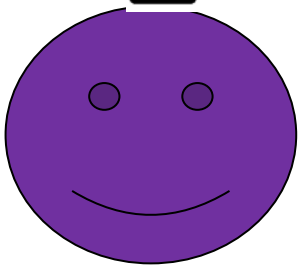
C_B

$M_B \cdot C_B$



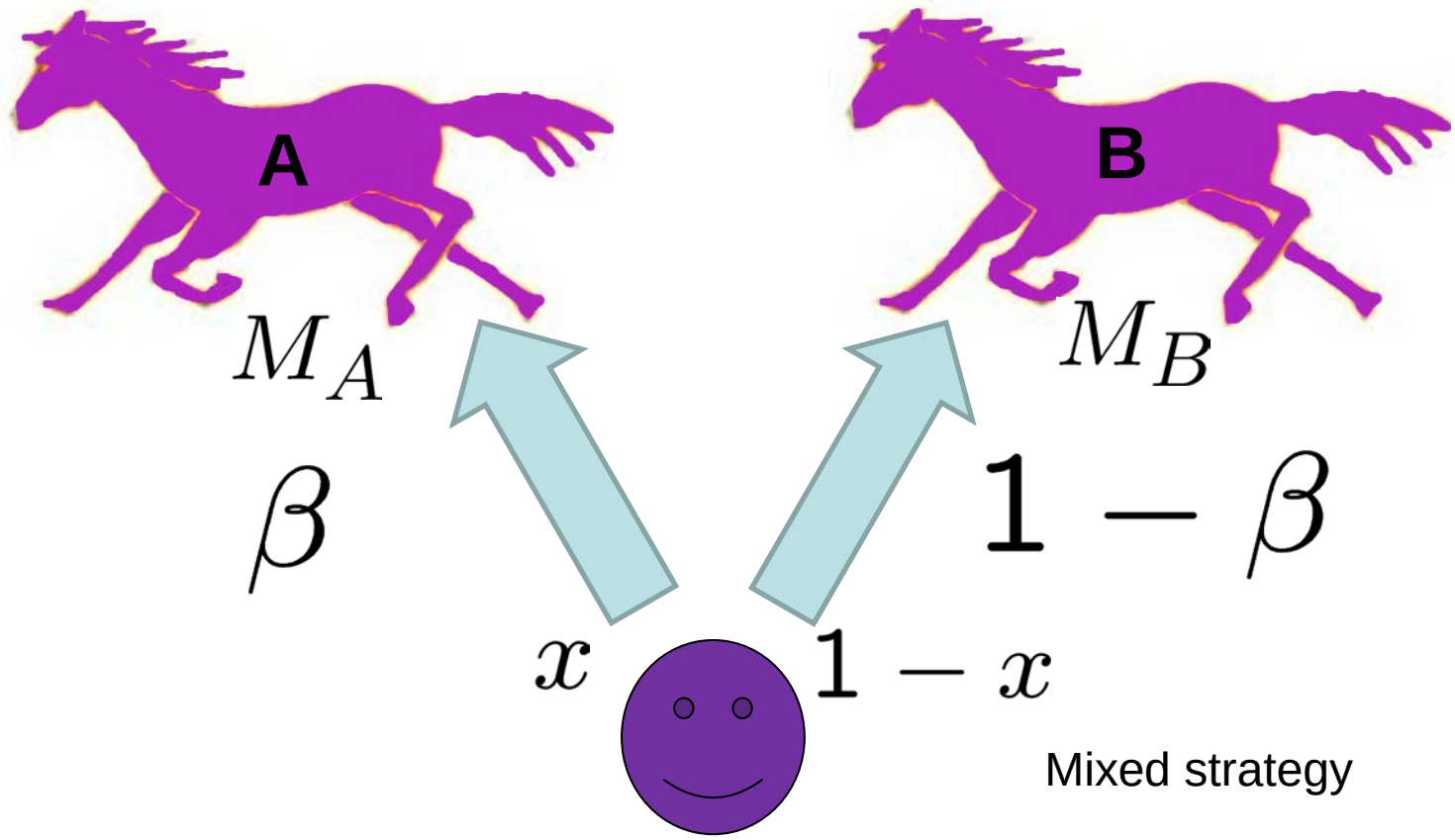
$$M_A = \frac{t+1}{C_A(t)+1}$$

1



$$M_B = \frac{t+1}{C_B(t)+1}$$

Optimal Strategy＝Maximization of Expected Return



Mixed strategy

Expected Return

$$R = x \cdot \beta \cdot M_A + (1-x) \cdot (1-\beta) \cdot M_B$$

Optimal Strategy＝Maximization of Expected Return

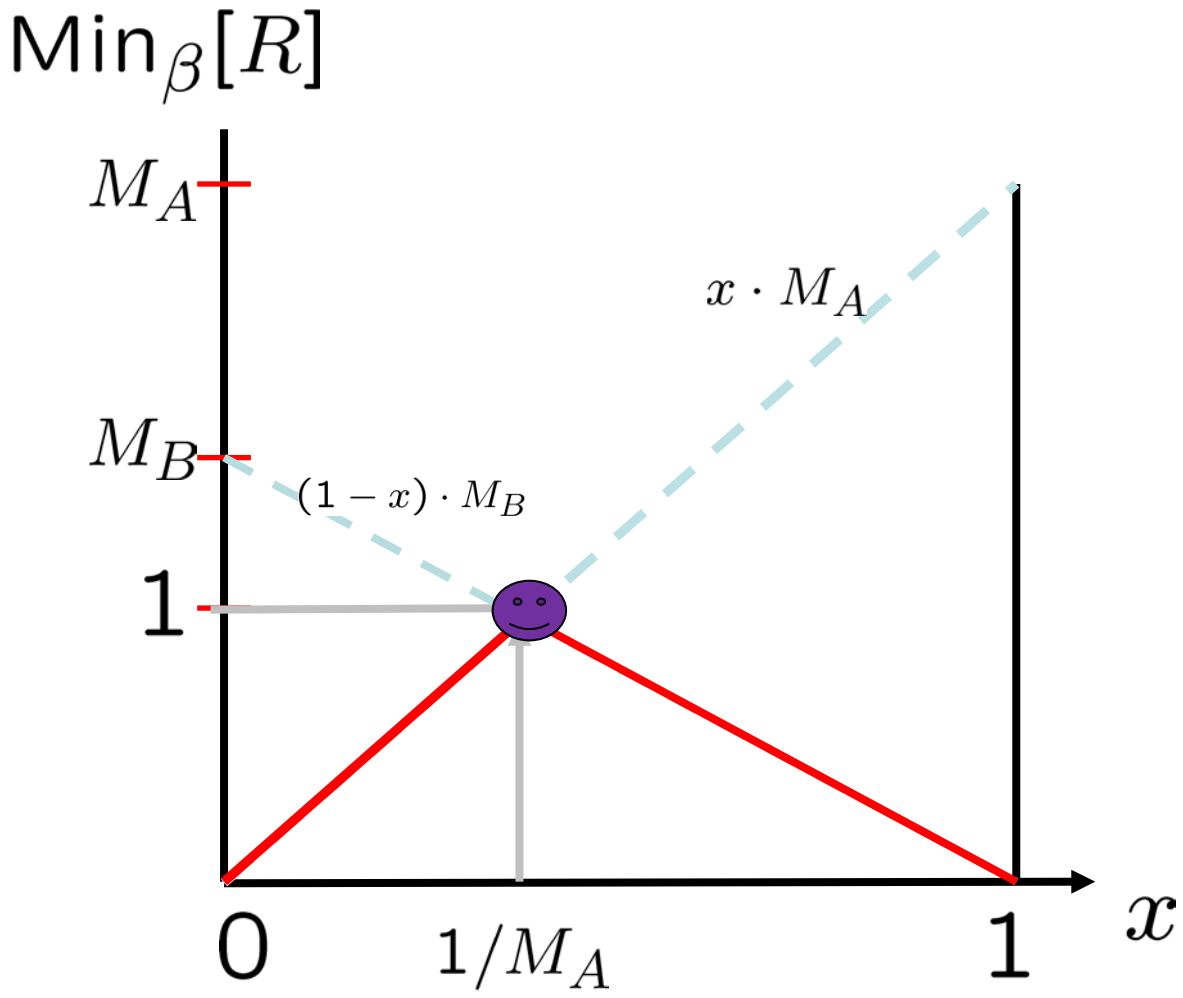
$$R = x \cdot \beta \cdot M_A + (1-x) \cdot (1-\beta) \cdot M_B$$
$$= \beta(x \cdot M_A - (1-x) \cdot M_B) + (1-x) \cdot M_B$$

Max-Min Strategy

Maximizaton of Minimum value of R

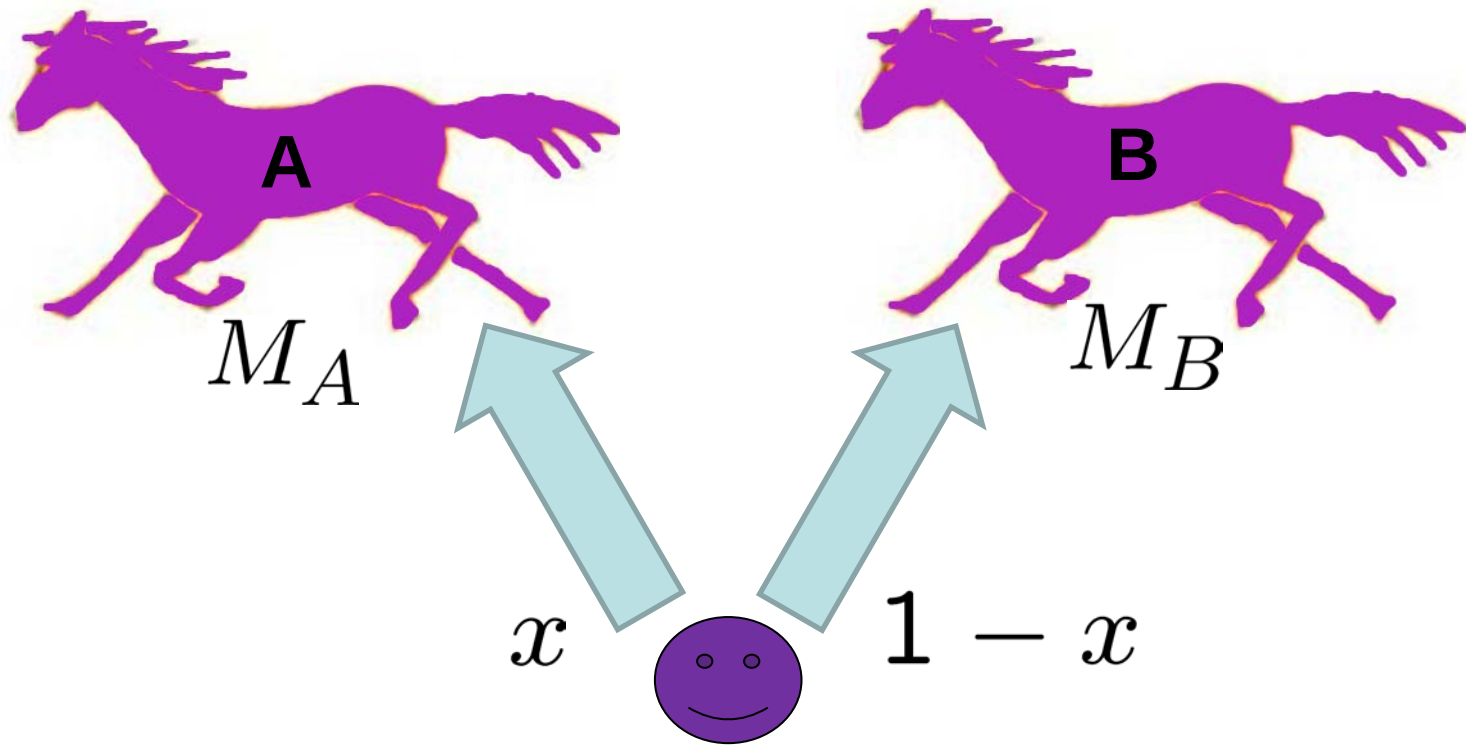
（= Minimization of Maximum value of Expected Loss）

$$\mathrm{Min}_{\beta}[R] = \begin{cases} (1-x) \cdot M_B & x \cdot M_A > (1-x) \cdot M_B \\ (1-x) \cdot M_B & x \cdot M_A = (1-x) \cdot M_B \\ x \cdot M_A & x \cdot M_A < (1-x) \cdot M_B \end{cases}$$



$$\mathrm{Max}_x(\mathrm{Min}_\beta[R])_x = 1 \text{ at } x = 1/M_A$$

Optimal Strategy＝Max-Min Strategy



$$x \cdot M_A = (1-x) \cdot M_B$$

$$M_A \propto \frac{1}{C_A} \quad \Rightarrow \quad x \propto C_A$$

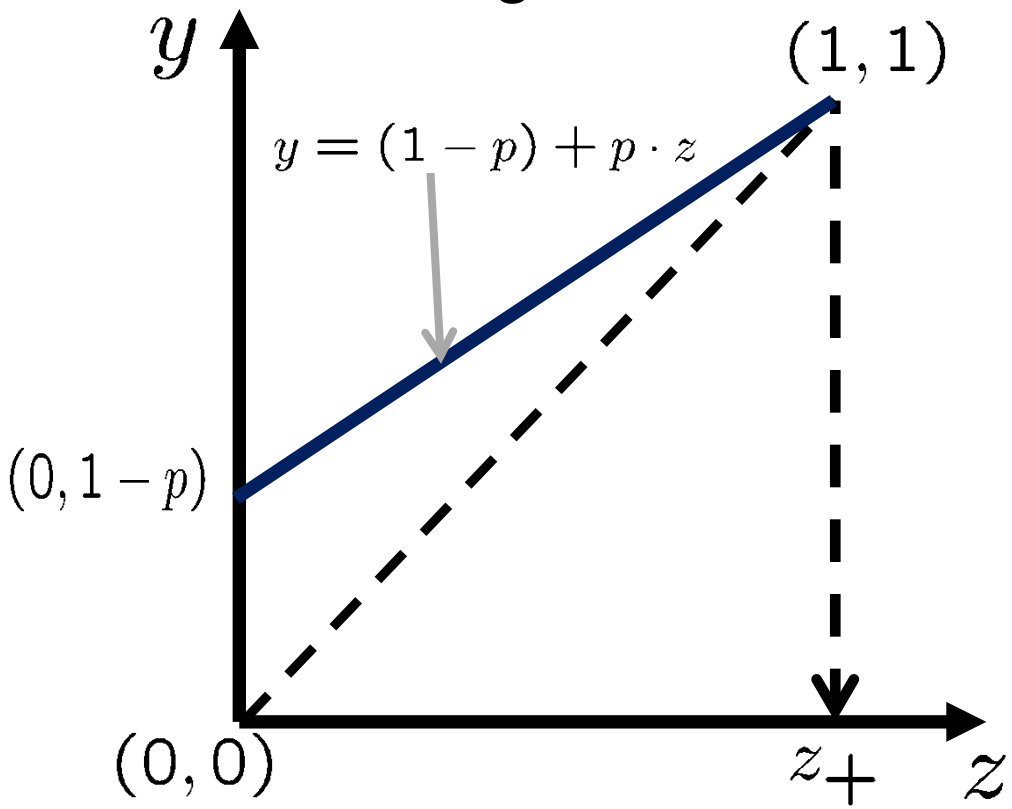
Analog Herder

Self-Consistent equation

$$z = (1-p)\cdot 1 + p\cdot q_h(t, t\cdot z) = q(z|p)$$

$$q_h(t, t\cdot z) = z$$

Analog Herder

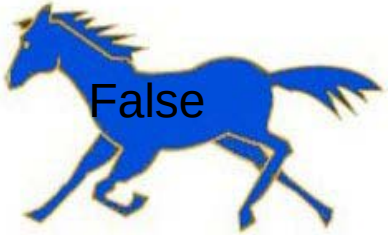


$$z_+ = 1 \text{ for } p < 1$$

Thermodynamic Limit of Zero-sum game

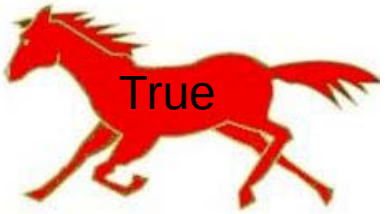
∞

$$\lim_{t\to\infty} C_A/t = 0$$

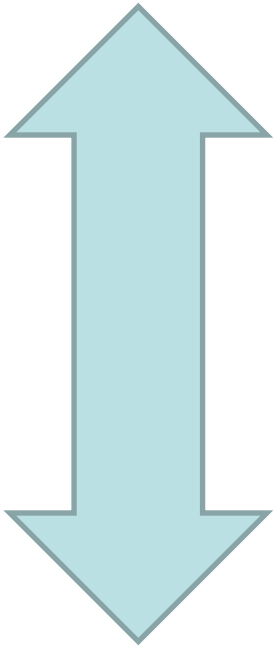


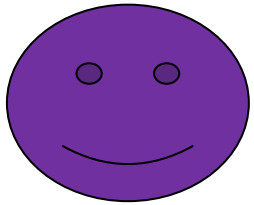
$$\lim_{t\to\infty} M_A = \infty$$

$$\lim_{t\to\infty} C_B/t = 1$$



$$\lim_{t\to\infty} M_B = 1$$

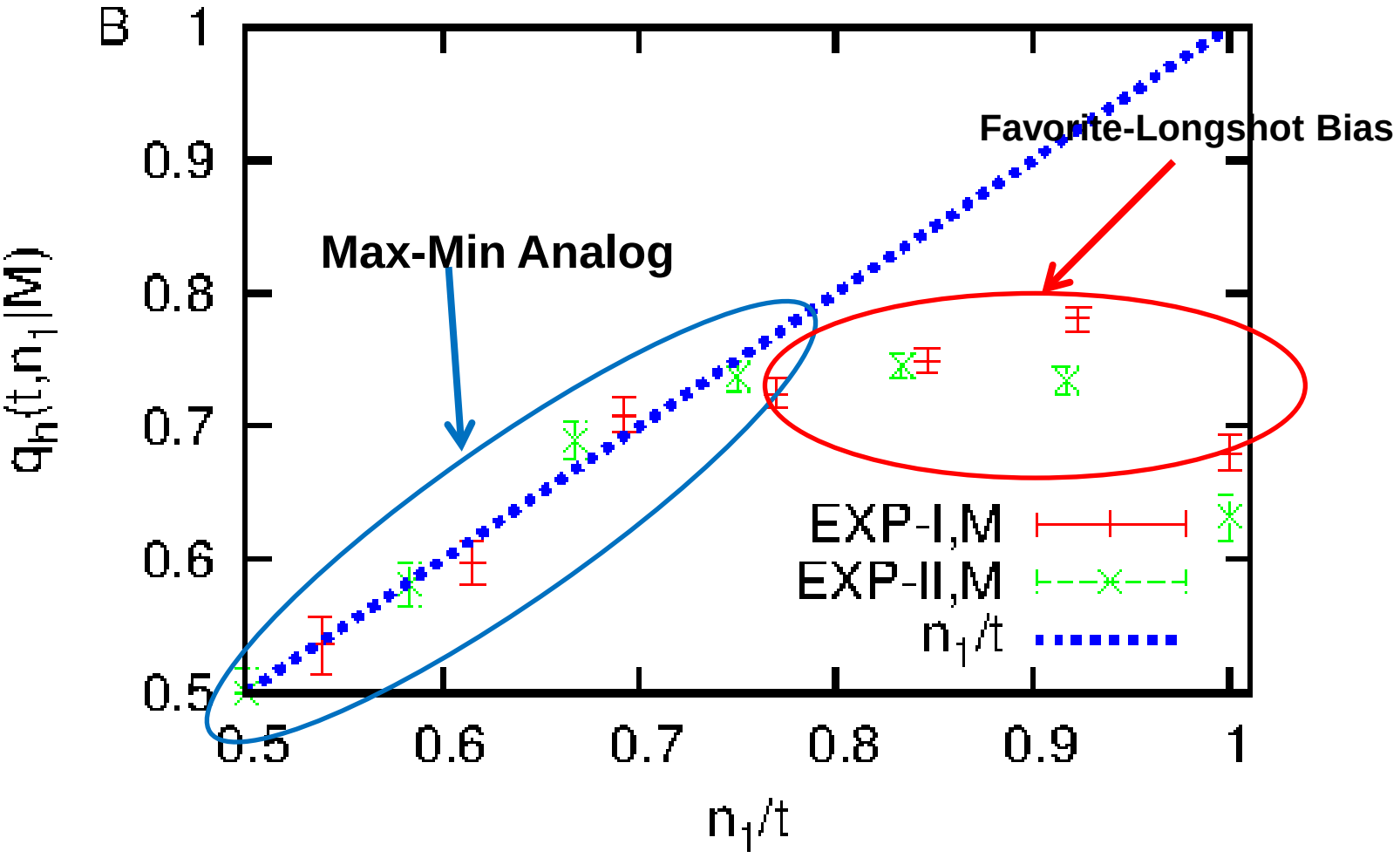




$$\infty + 1$$

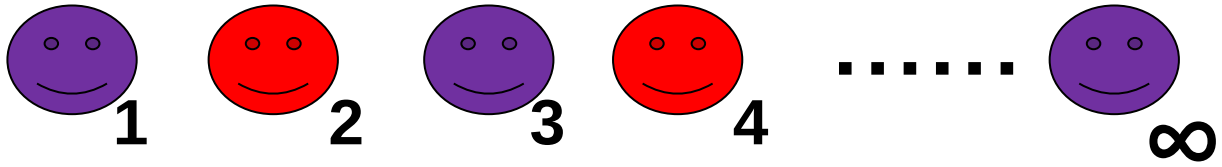
$q_h(t, n_1)$: Herder's Response Function

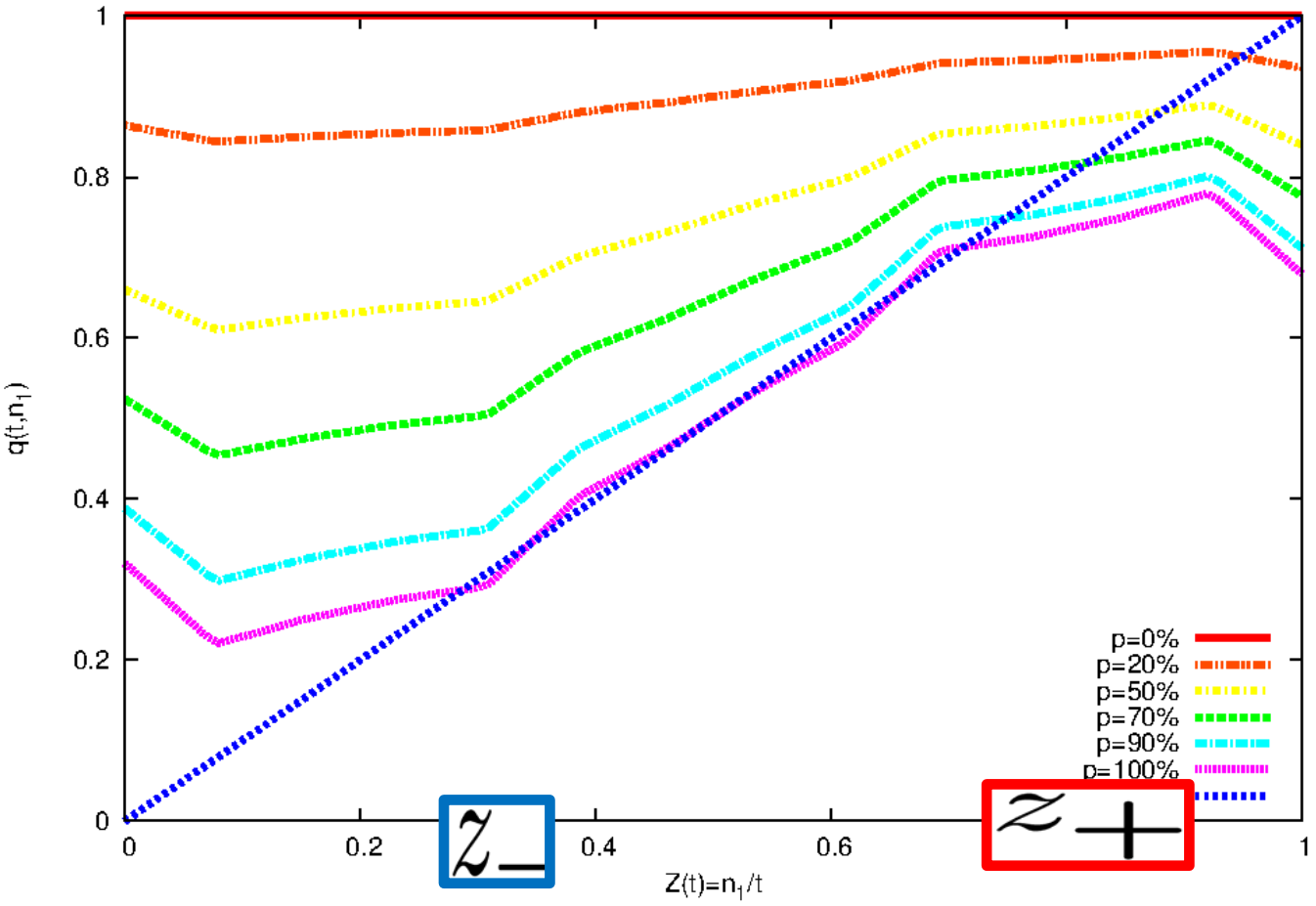
その選択肢を選ぶ確率



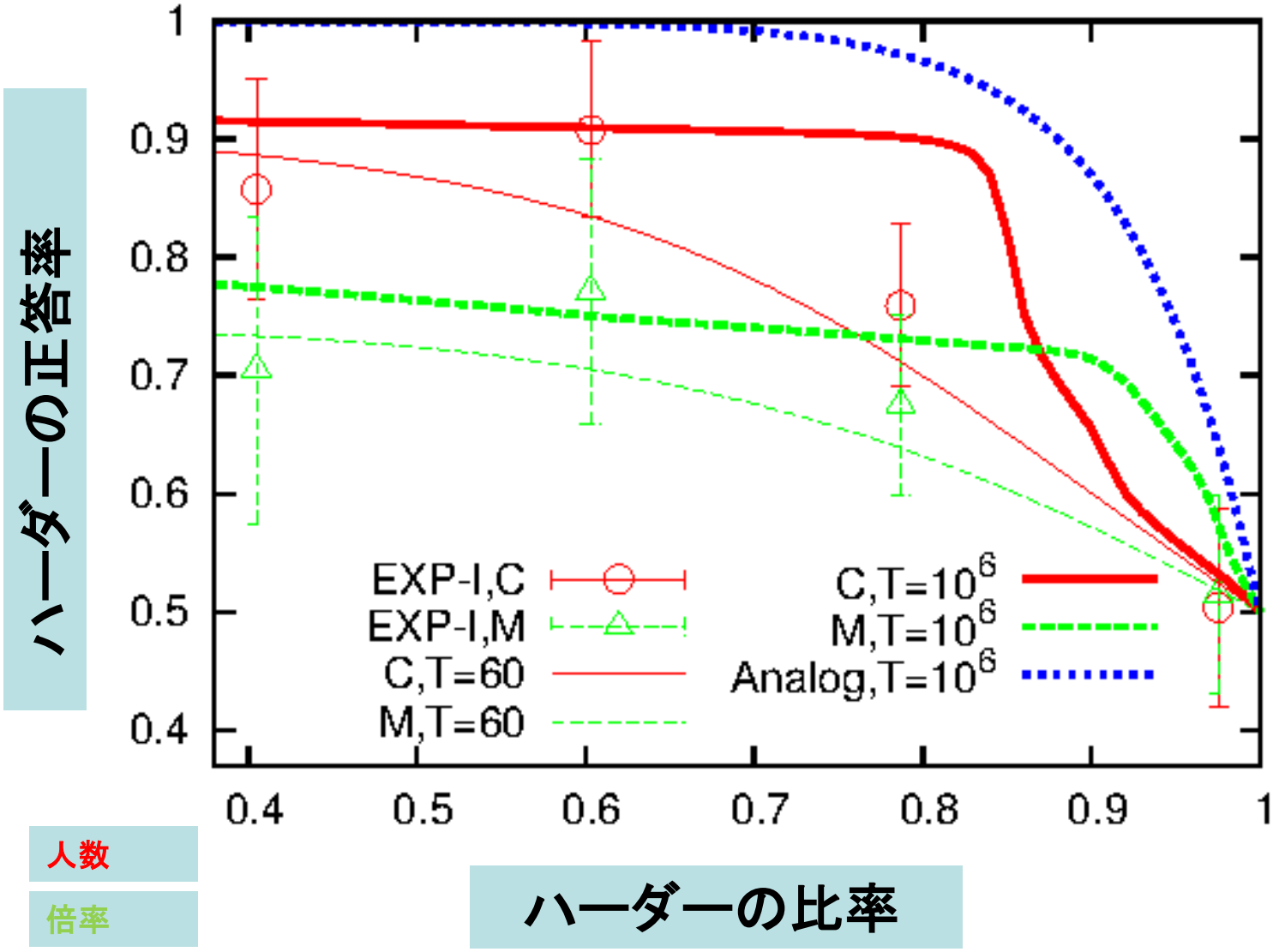
ある選択肢を選んだ比率

$p < 100\%$





$$p_c = 96\%$$


ハーダーの正答率
1
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.5
0.6
0.7
0.8
0.9
1
EXP-I,C
EXP-I,M
C,T=60
M,T=60
C,T=10^6
M,T=10^6
Analog,T=10^6
人数
倍率
ハーダーの比率

Summary of Experimental Results

Microscopic Level

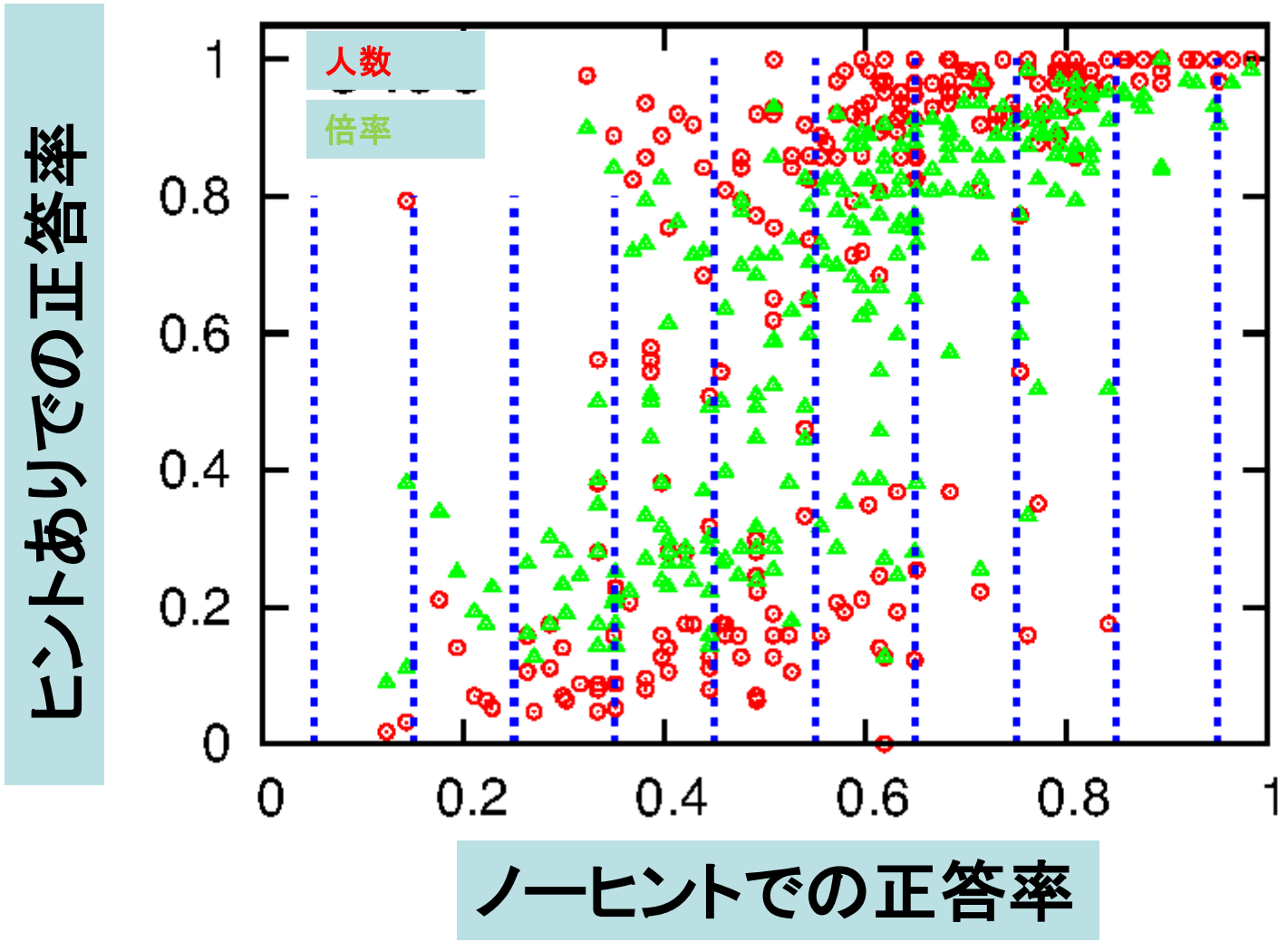
Max-Min Strategy 4/3<m<4 & Bias for m<4/3 & m>4

Macroscopic Level

Herder’s % of Correct Choice is not so high by the bias.

Reference

Collective Adoption of Max-Min Strategy in an information cascade voting experiment
S.Mori, M. Hisakado and T. Takahashi, arXiv:1211.3193


ヒントありでの正答率
人数
倍率
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
ノーヒントでの正答率

Voting Experiment

No. Info.

Q.30:Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky	B : Beethoven
○	○

Answer

Voting Experiment

All previous subjects' Info.

Up to now 7 subjects have answered.

Their choices are as follows. Please choose.

Q.30:Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky	B : Beethoven
1人	6人
○	○

Answer

Voting Experiment

Odds Info.

Up to now 10 subjects have answered.

10

Their choices are given as Odds as folows.

If your choise is true, the points earned is multipled by the Odds.

Even if a choice with large odds is more likely to be wrong, it is rational to choose it with the objective of expected ponit. Please choose.

Q.30:Which composer is famous for the Symphonie No.6 Pathetique ?

A : Beethoven	B : Tchaikovsky
×1.4	×2.8
○	○

7

3

$\frac{10+1}{7+1}$

$\frac{10+1}{3+1}$

Point is multiplied by Odds