

2択クイズの情報カスケード実験とゲーム論的最適戦略

ヒトはオッズにどう反応するのか？

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問題: ロダンの「考える人」が肘をついている場所はどこ？

A: 右脚

B: 左脚

ヒント: 人数

A: 4人

B: 6人

Return=1



ヒント: 倍率

A: 10/4 倍

B: 10/6 倍

A: 11/5 倍

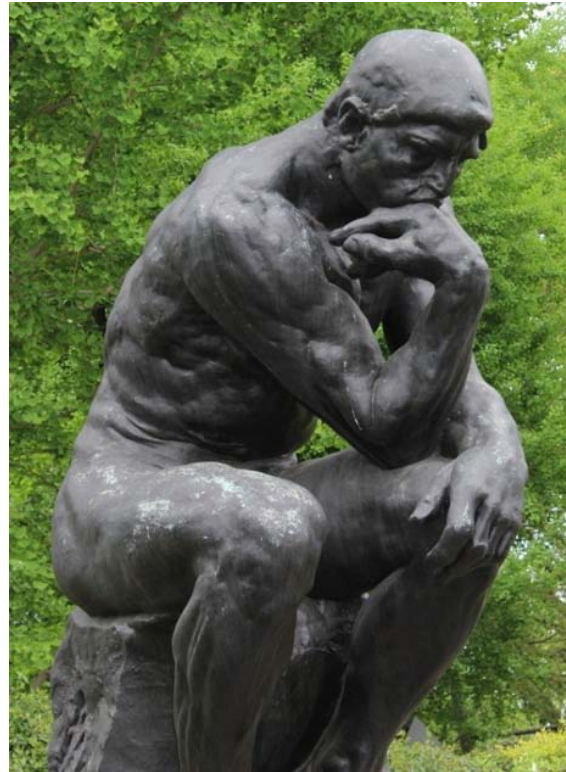
B: 11/7 倍

Return = 倍率

正解:B:左脚

ノーヒントでの正答率

グループA $T = 57$	53%
グループB $T = 63$	46%

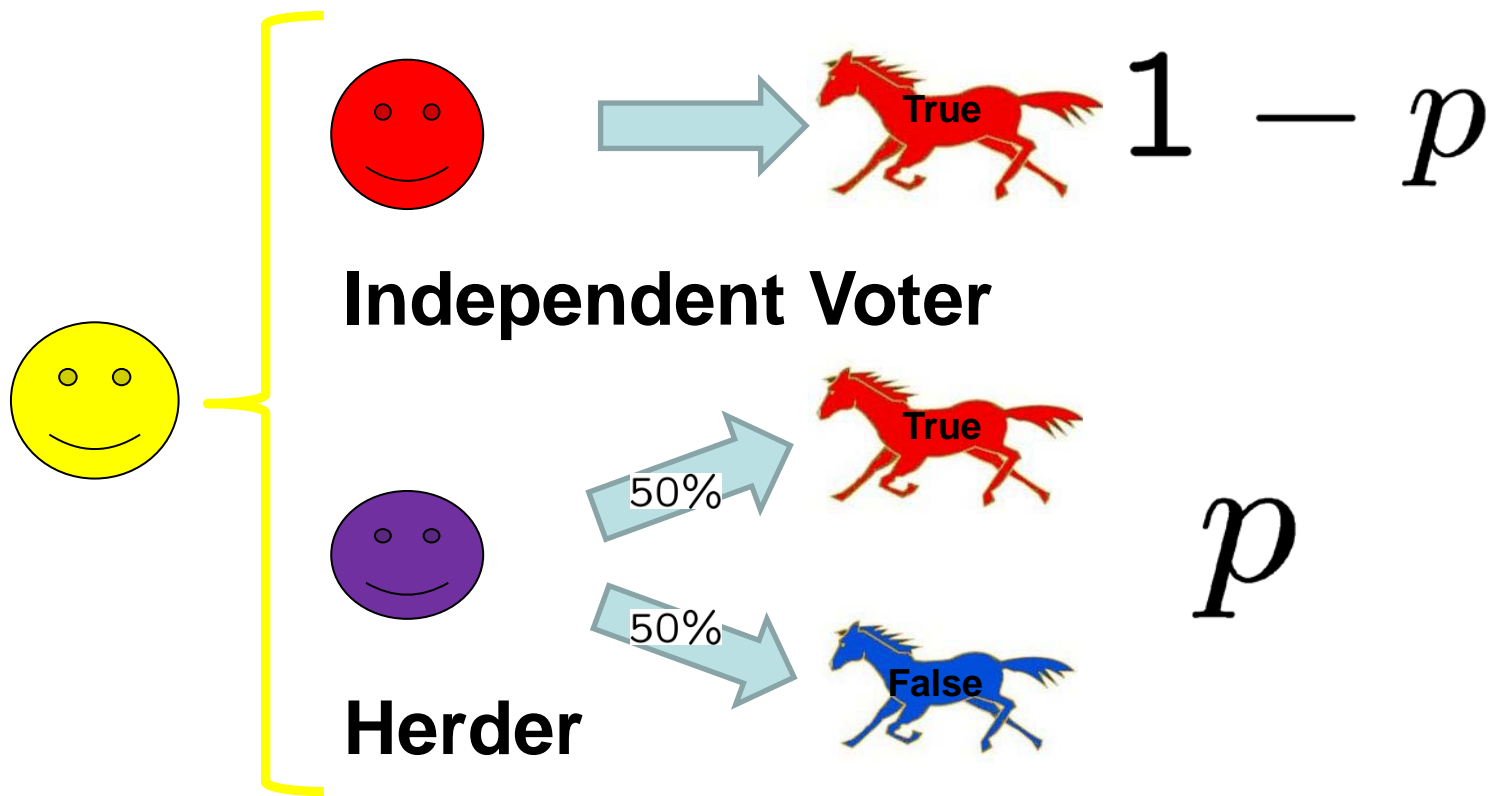


ヒント:人数での正答率

グループA	86%
グループB	16%

ヒント:倍率での正答率

グループA	74%
グループB	40%



平均正答率

$$q = (1 - p) \cdot 1 + p \cdot \frac{1}{2} = 1 - \frac{1}{2}p$$

ヒント:人数



どう選択するのか？

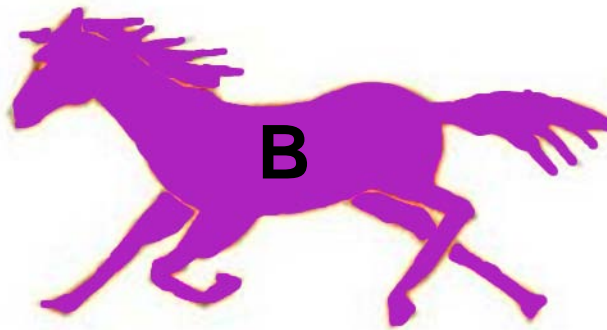
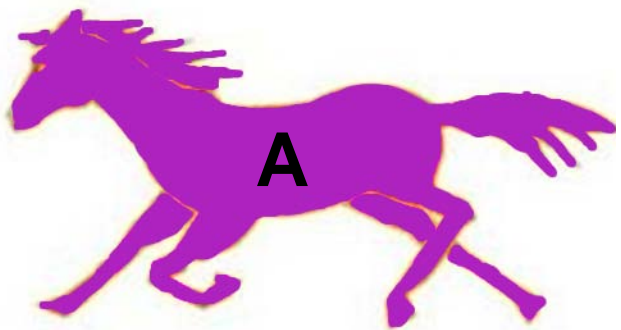


t 人

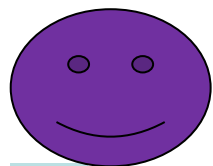
$$\{C_A(t), C_B(t)\}$$



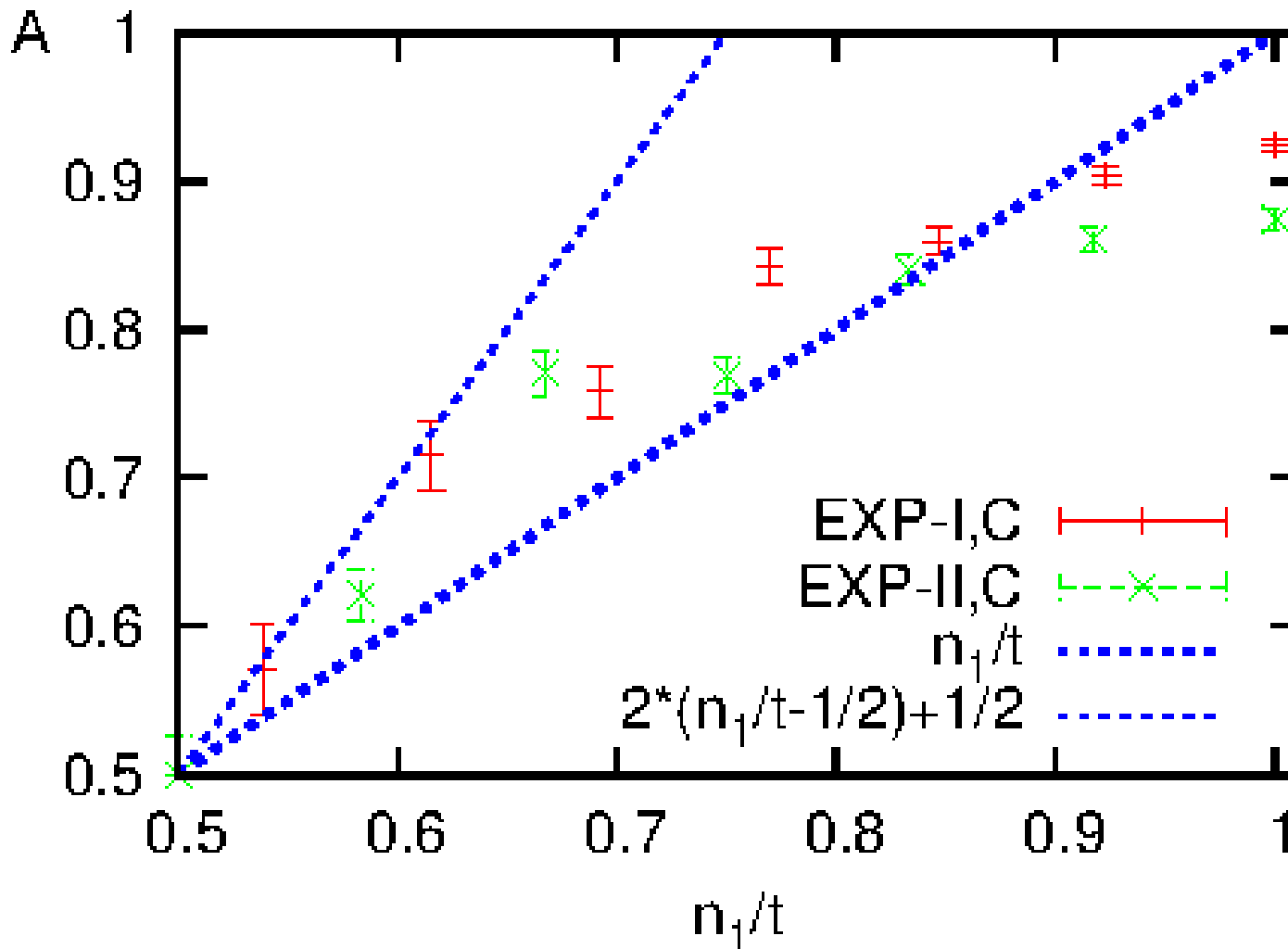
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$q_h(t, n_1)$: Herder's Response Function



その選択肢を選ぶ確率



ある選択肢を選んだ人の比率

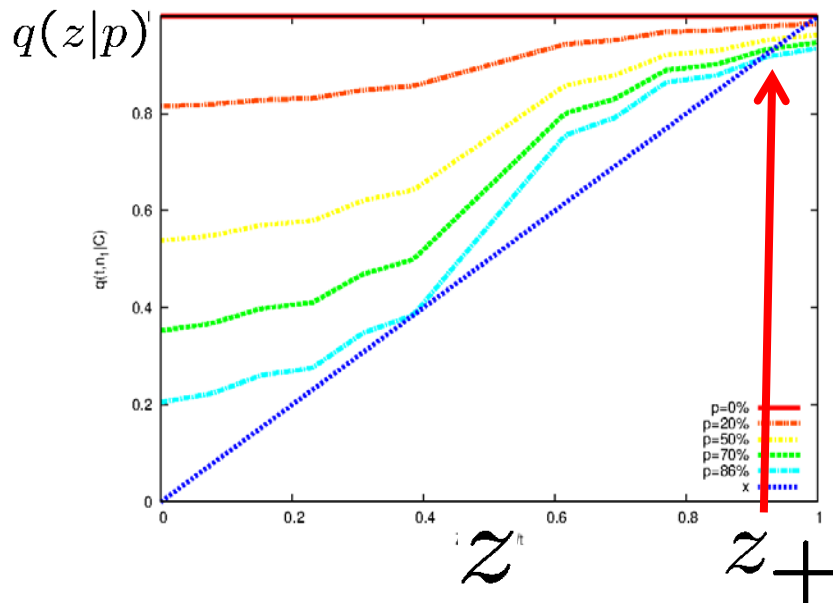
Self-Consistent equation

$$z = (1 - p) \cdot 1 + p \cdot q_h(t, t \cdot z) = q(z|p)$$

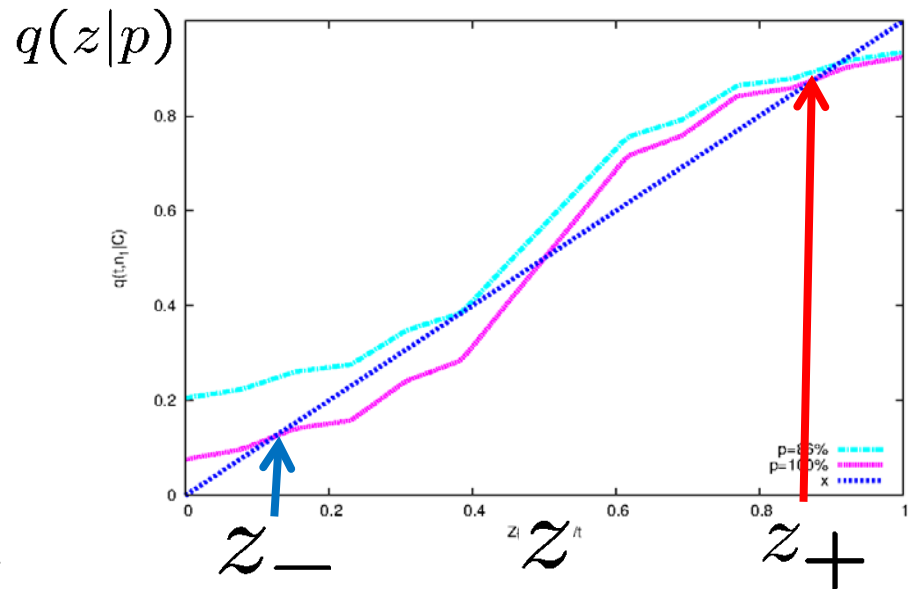


$p \leq p_c = 86\%$

$p \geq p_c = 86\%$



One-Peak Phase



Two-Peak Phase

Information Cascade Phase transition

ヒント:倍率

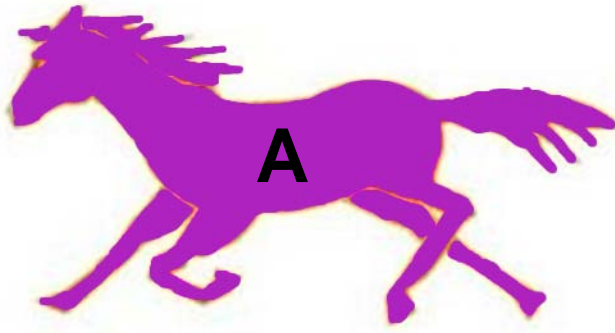
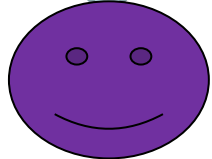


$$\{C_A(t), C_B(t)\}$$

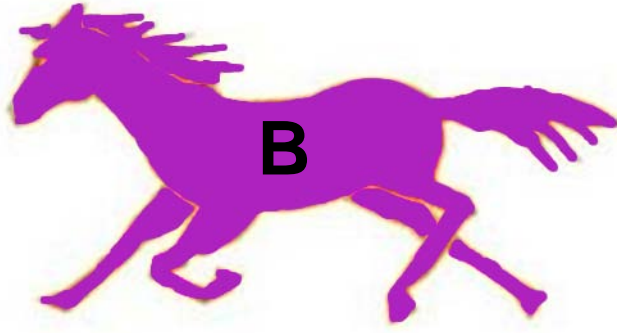
Zero-sum game

$$M_A = \frac{t + 1}{C_A(t) + 1}$$

$$M_B = \frac{t + 1}{C_B(t) + 1}$$



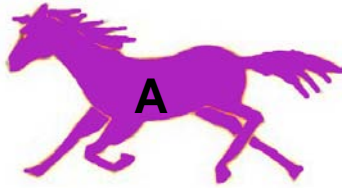
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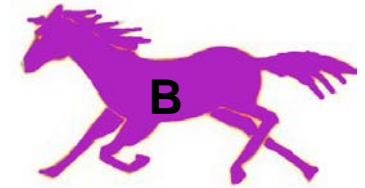
Zero-sum game



$$C_A$$
$$M_A \cdot C_A$$



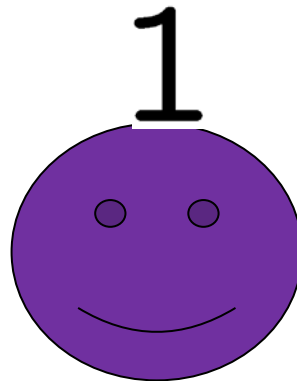
$$C_B$$
$$M_B \cdot C_B$$



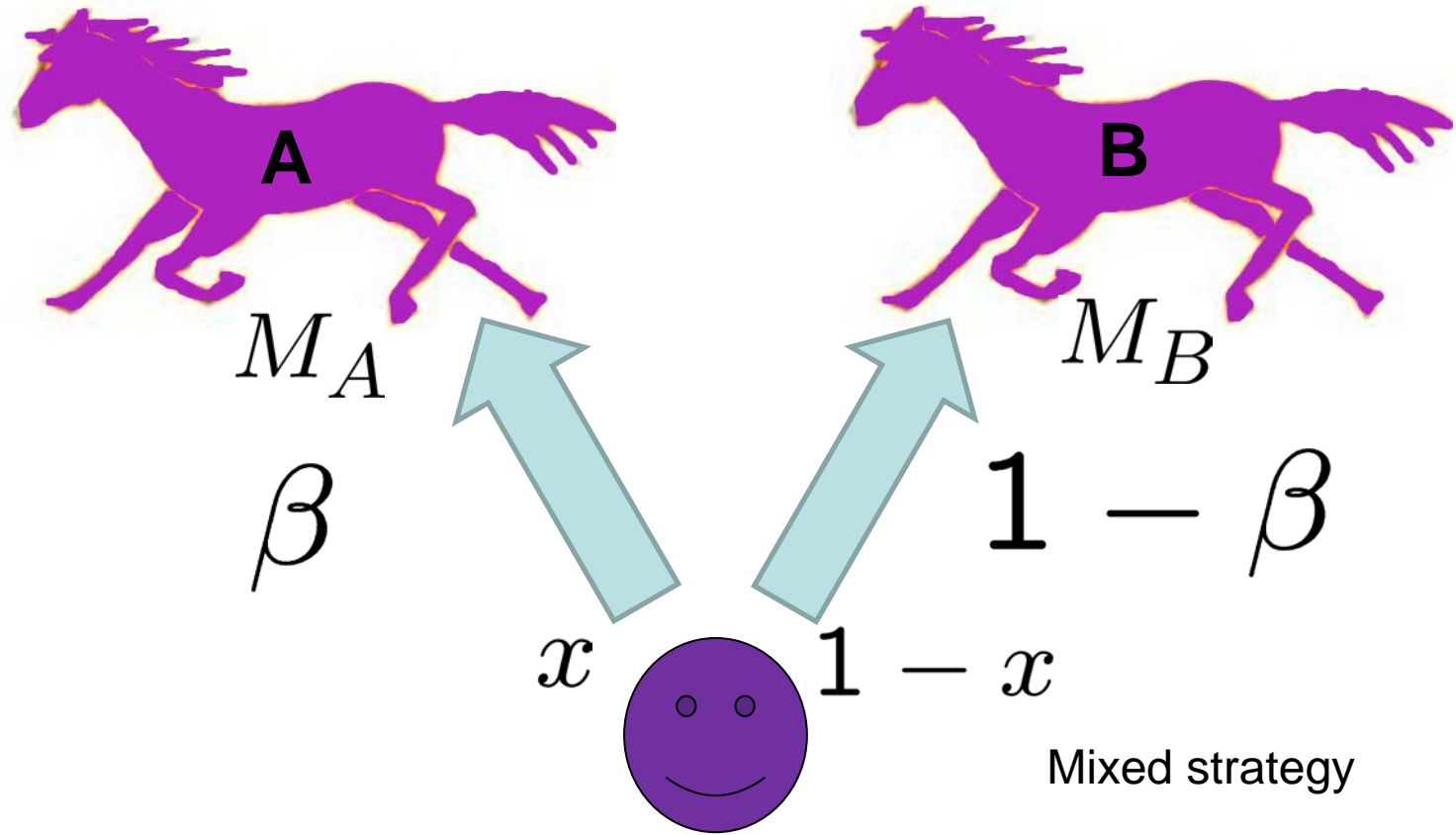
$$C_A + C_B + 1$$

$$M_A = \frac{t + 1}{C_A(t) + 1}$$

$$M_B = \frac{t + 1}{C_B(t) + 1}$$



Optimal Strategy = Maximization of Expected Return



Expected Return

$$R = x \cdot \beta \cdot M_A + (1 - x) \cdot (1 - \beta) \cdot M_B$$

Optimal Strategy = Maximization of Expected Return

$$\begin{aligned} R &= x \cdot \beta \cdot M_A + (1 - x) \cdot (1 - \beta) \cdot M_B \\ &= \beta(x \cdot M_A - (1 - x) \cdot M_B) + (1 - x) \cdot M_B \end{aligned}$$

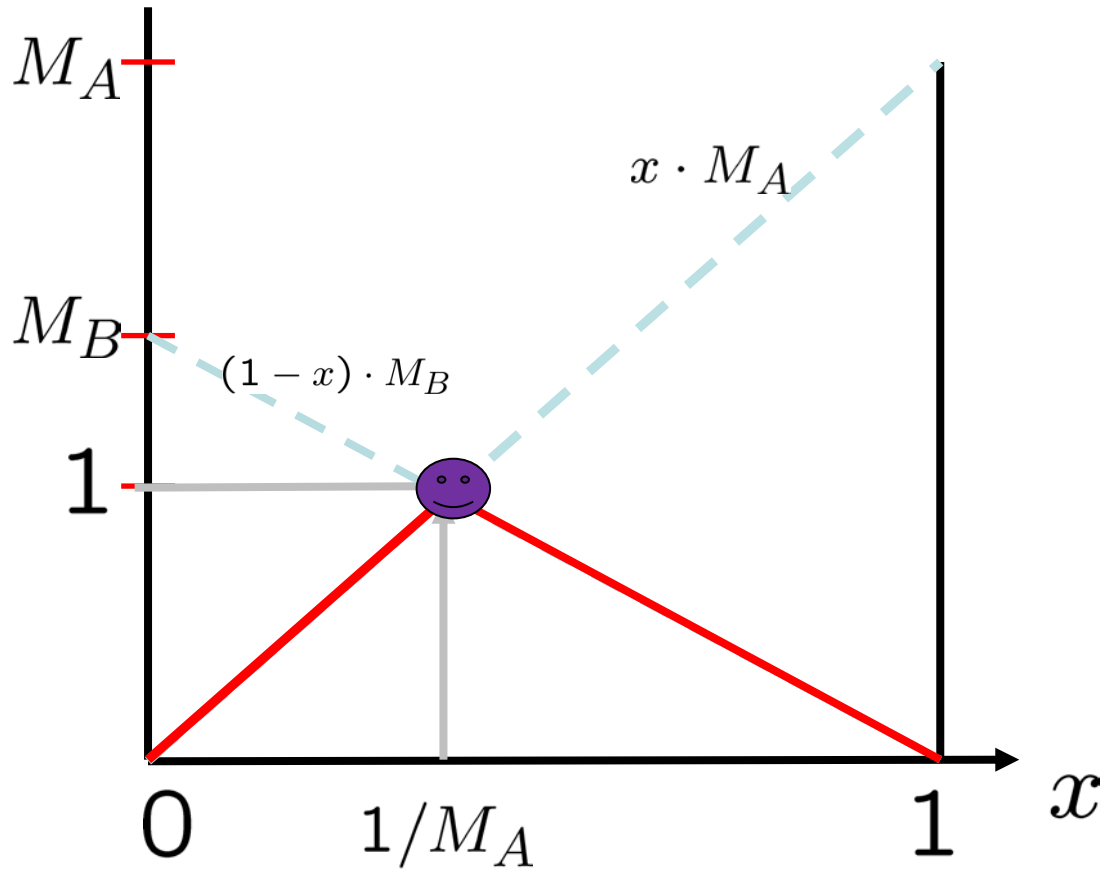
Max-Min Strategy

Maximization of Minimum value of R

(= Minimization of Maximum value of Expected Loss)

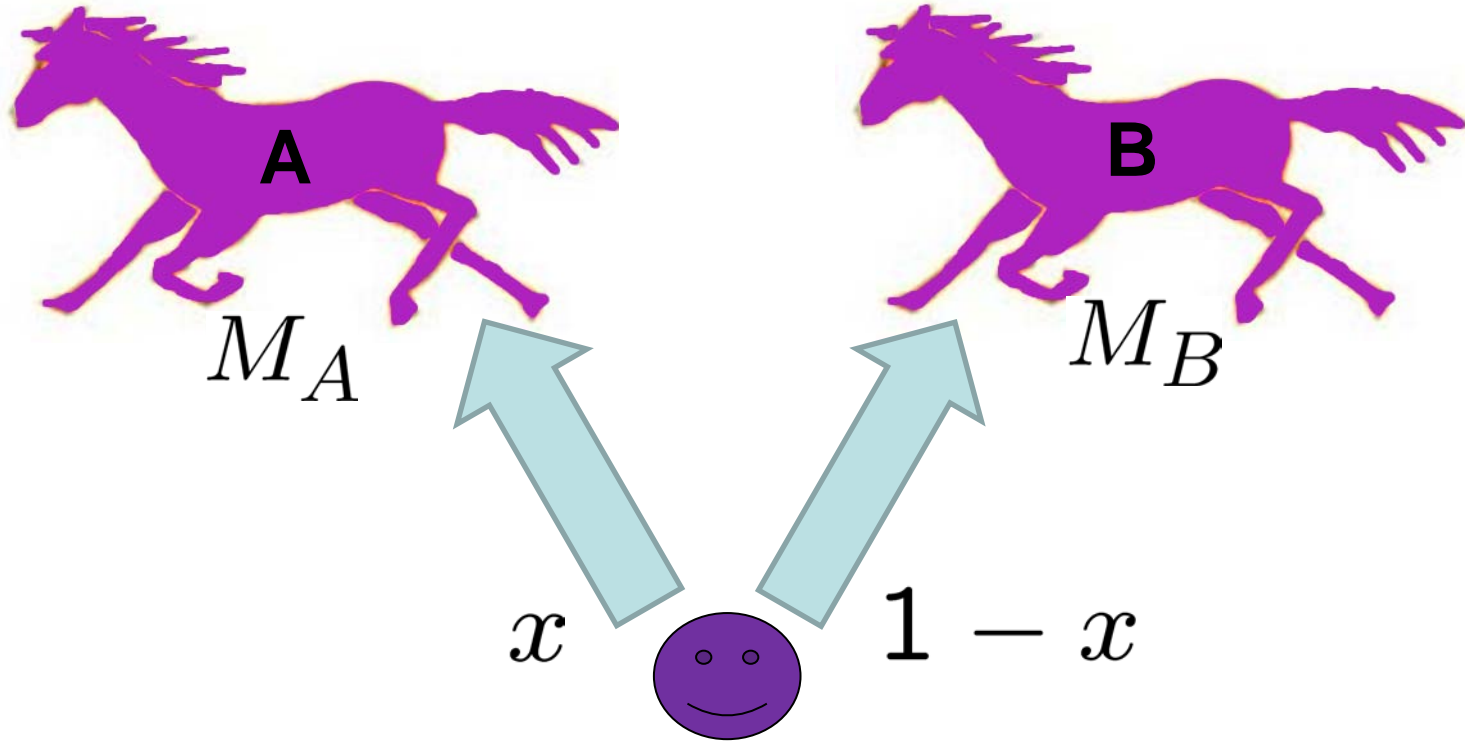
$$\text{Min}_{\beta}[R] = \begin{cases} (1 - x) \cdot M_B & x \cdot M_A > (1 - x) \cdot M_B \\ (1 - x) \cdot M_B & x \cdot M_A = (1 - x) \cdot M_B \\ x \cdot M_A & x \cdot M_A < (1 - x) \cdot M_B \end{cases}$$

$\text{Min}_\beta[R]$



$$\text{Max}_x(\text{Min}_\beta[R])_x = 1 \text{ at } x = 1/M_A$$

Optimal Strategy = Max-Min Strategy



$$x \cdot M_A = (1 - x) \cdot M_B$$

$$M_A \propto \frac{1}{C_A}$$

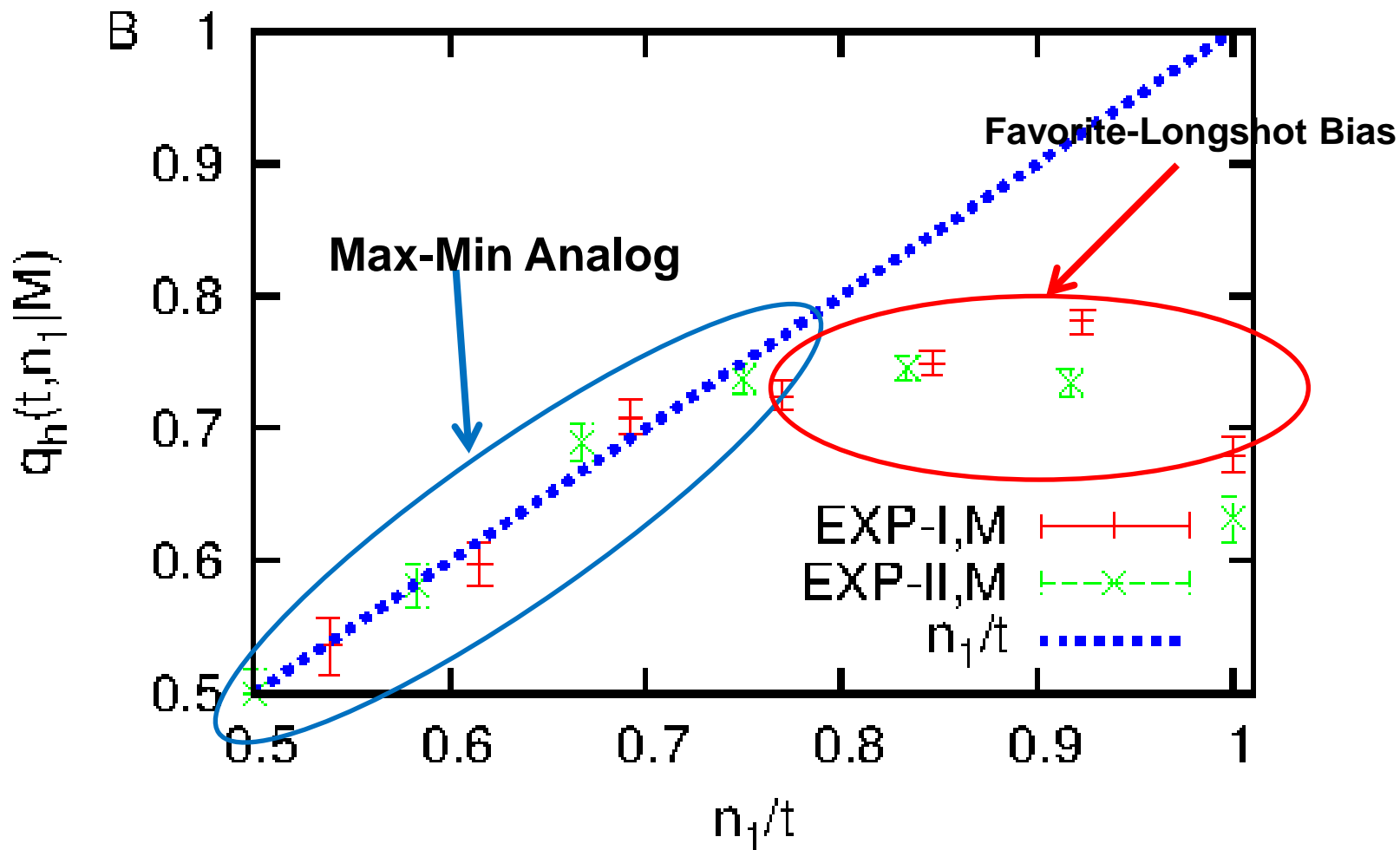


$$x \propto C_A$$

Analog Herder

$q_h(t, n_1)$: Herder's Response Function

その選択肢を選ぶ確率



ある選択肢を選んだ比率

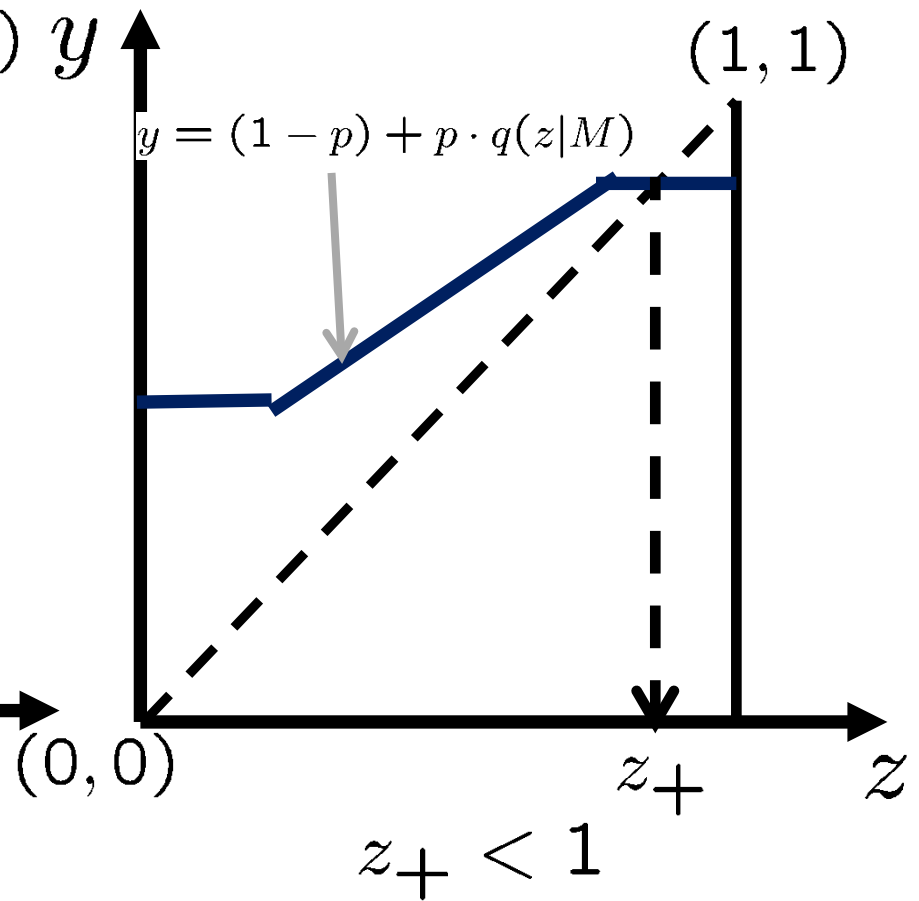
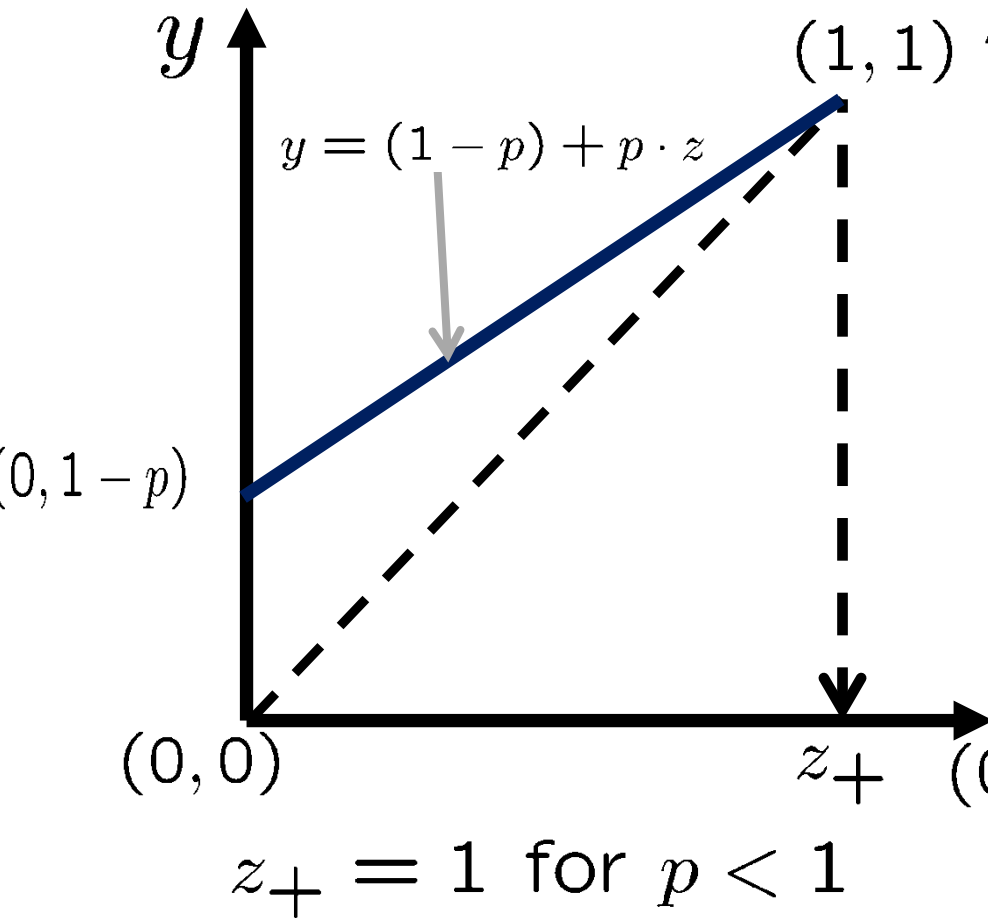
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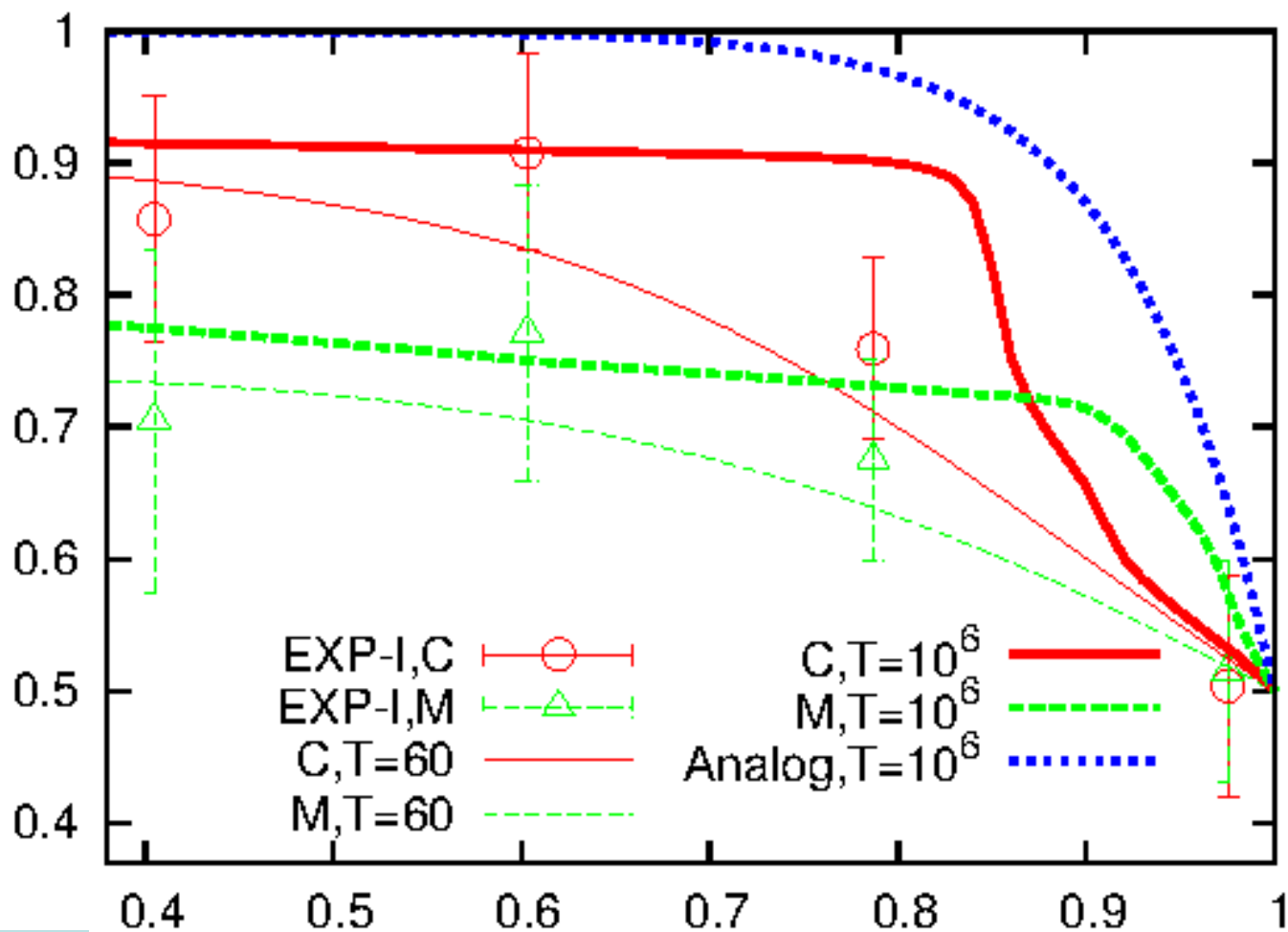


$$q_h(t, t \cdot z) = z$$

Analog Herder



ハーダーの正答率



人数

倍率

ハーダーの比率

Summary of Experimental Results

Microscopic Level

Max-Min Strategy $4/3 < m < 4$ & Bias for $m < 4/3$ & $m > 4$

Macroscopic Level

Herder's % of Correct Choice is not so high by the bias.

Reference

Collective Adoption of Max-Min Strategy in an information cascade voting experiment
S.Mori, M. Hisakado and T. Takahashi, arXiv:1211.3193