

# How one choose with Multipliers ?

倍率情報下での最適な情報のコピーと実験による検証

Experimental test of adoption of optimal strategy for herd when payoffs have multipliers



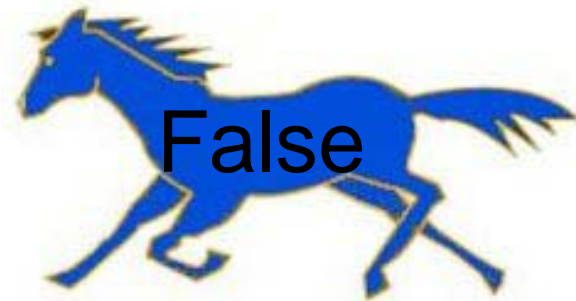
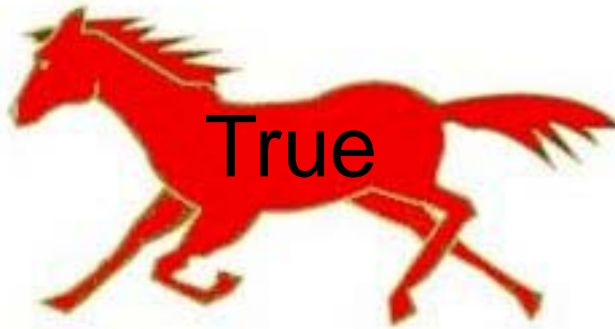
**S.Mori, Kitasato University**

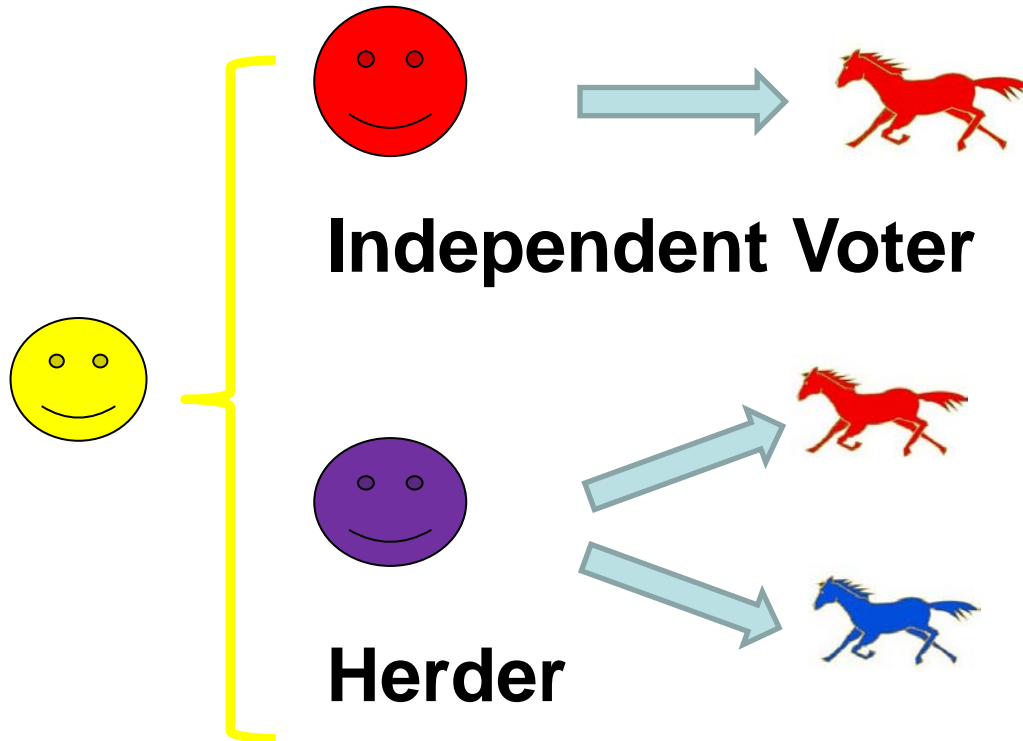
**Collaborators**

**M.Hisakado, Standard and Poor's**

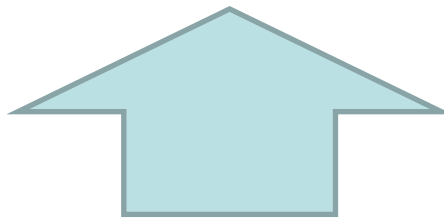
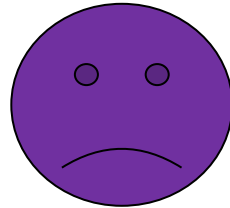
**T.Takahashi, Hokkaido University**

# Quiz : A or B





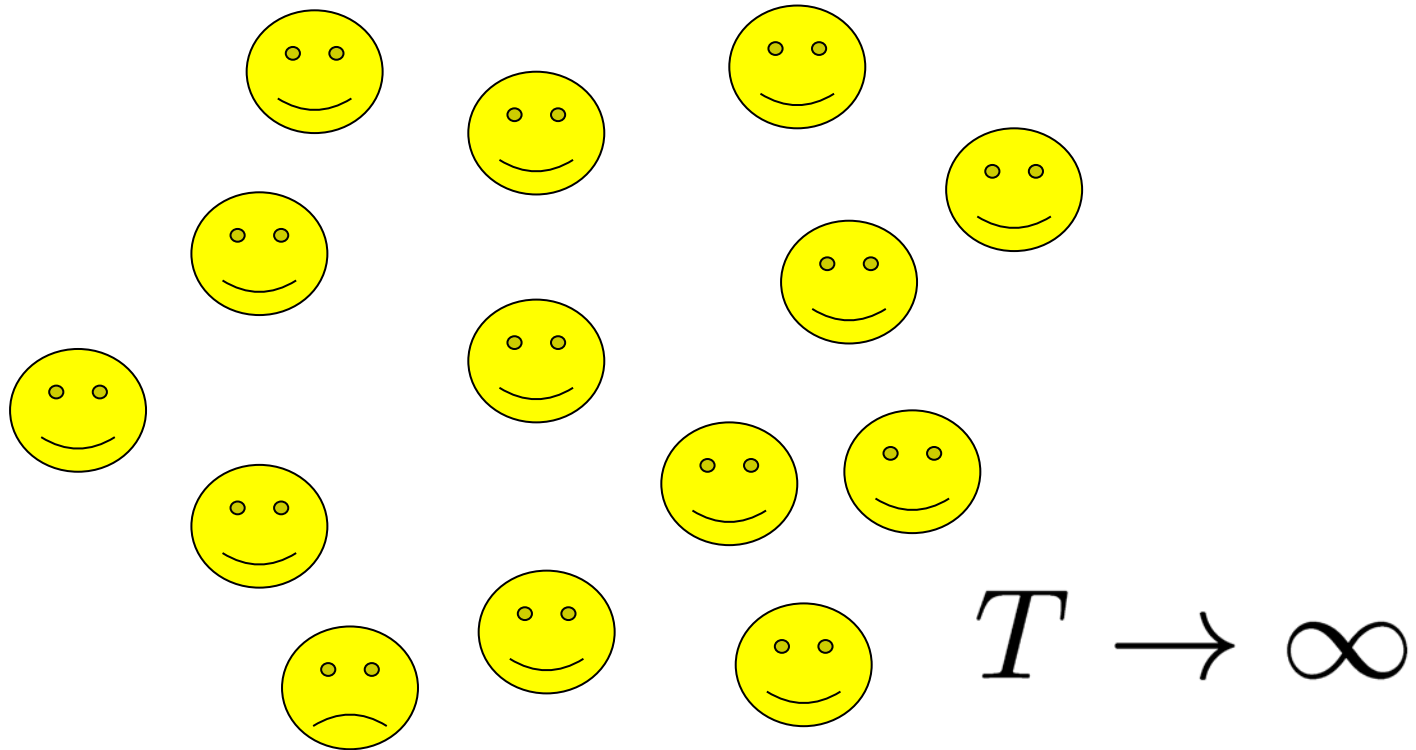
Q. How he chooses ?



**Others' Choices**

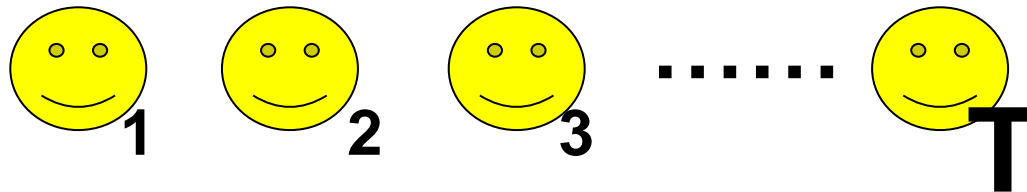
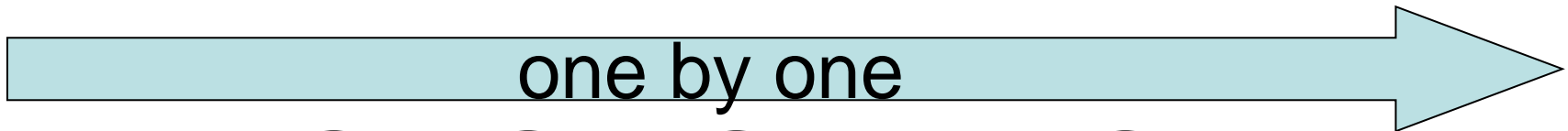
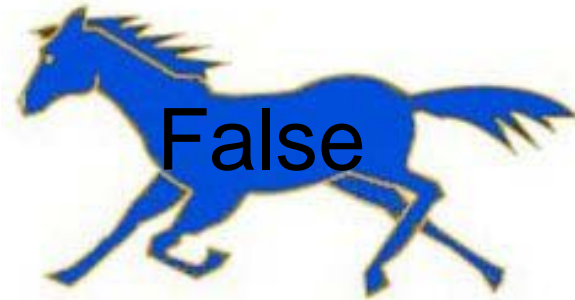
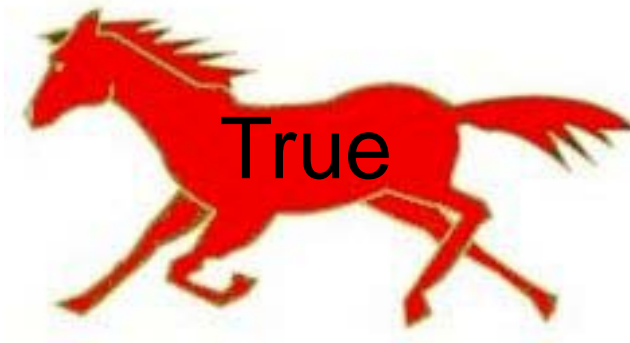


Q. What happens  
at the macroscopic level ?



# Experimental Setup

# Quiz : A or B



**EXP-I :  $T_{avg}=60,2$  Groups,120 questions @Hokkaido Univ.**

**EXP-II :  $T_{avg}=50$  ,2 Groups,120 questions @Hokkaido Univ.**

Q: On which thigh does the Thinker of Rodin rest his elbow ?

## No Hint

A	B
Left	Right

2 Pts



### Case C

A	B
10	20
Left	Right

2 Pts

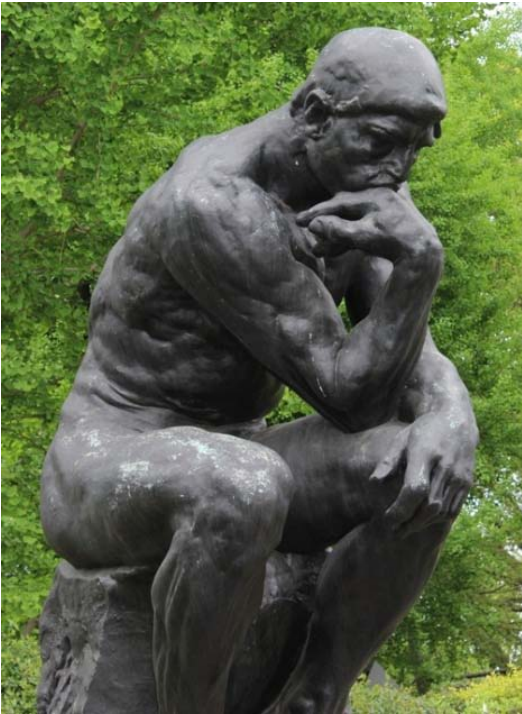
### Case M

A	B
31/11	31/21
Left	Right

Pts=Multiplier



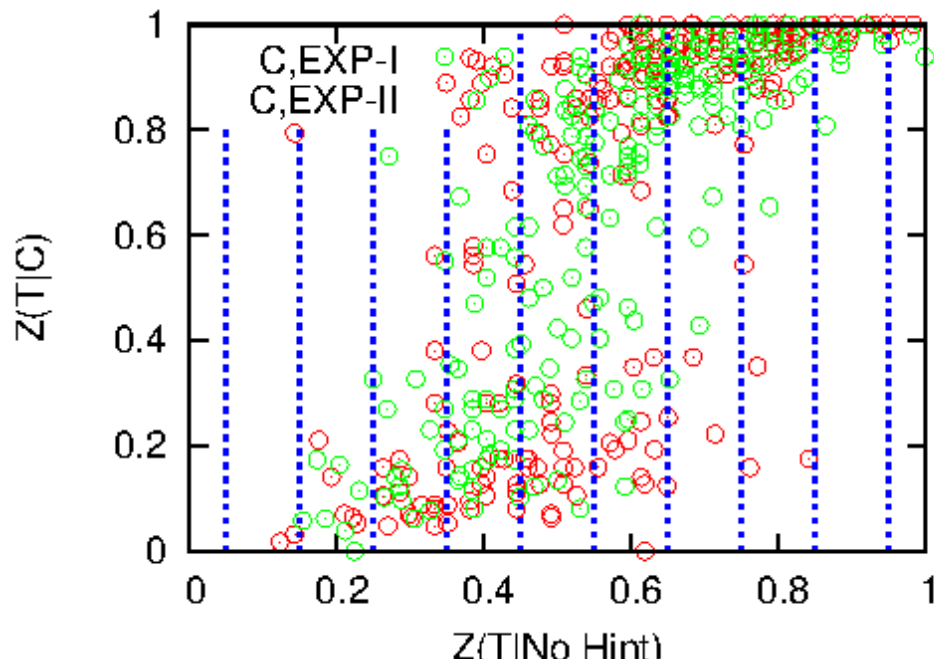
# Ans. A: Left



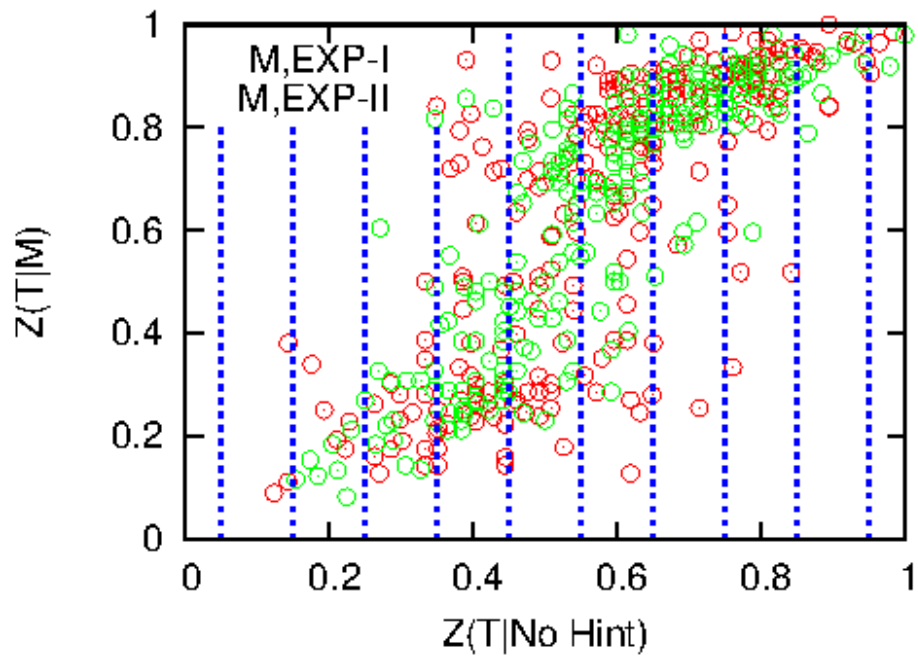
<b>% of Correct Ans.</b>	<b>Group I</b>	<b>Group II</b>
Z(T  No Hint)	53%	46%
Z(T C)	86%	16%
Z(T M)	74%	40%

From EXP-I

# Case C



# Case M



# Q. How he copies ?

Case C



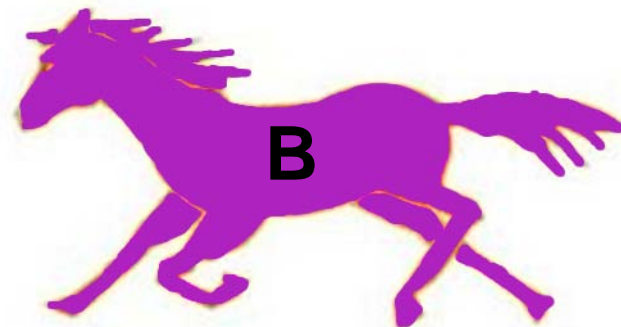
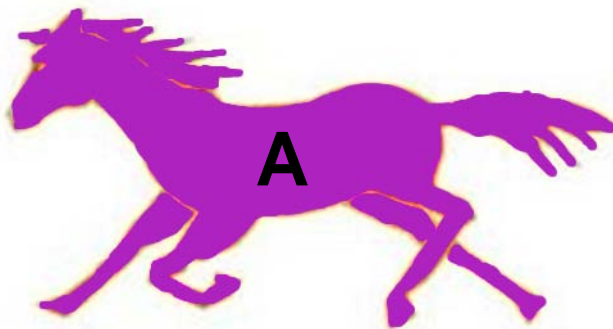
$$\{C_A(t), C_B(t)\}$$



$$q_h(C_A(t)/t|C)$$

$$q_h(C_B(t)/t|C)$$

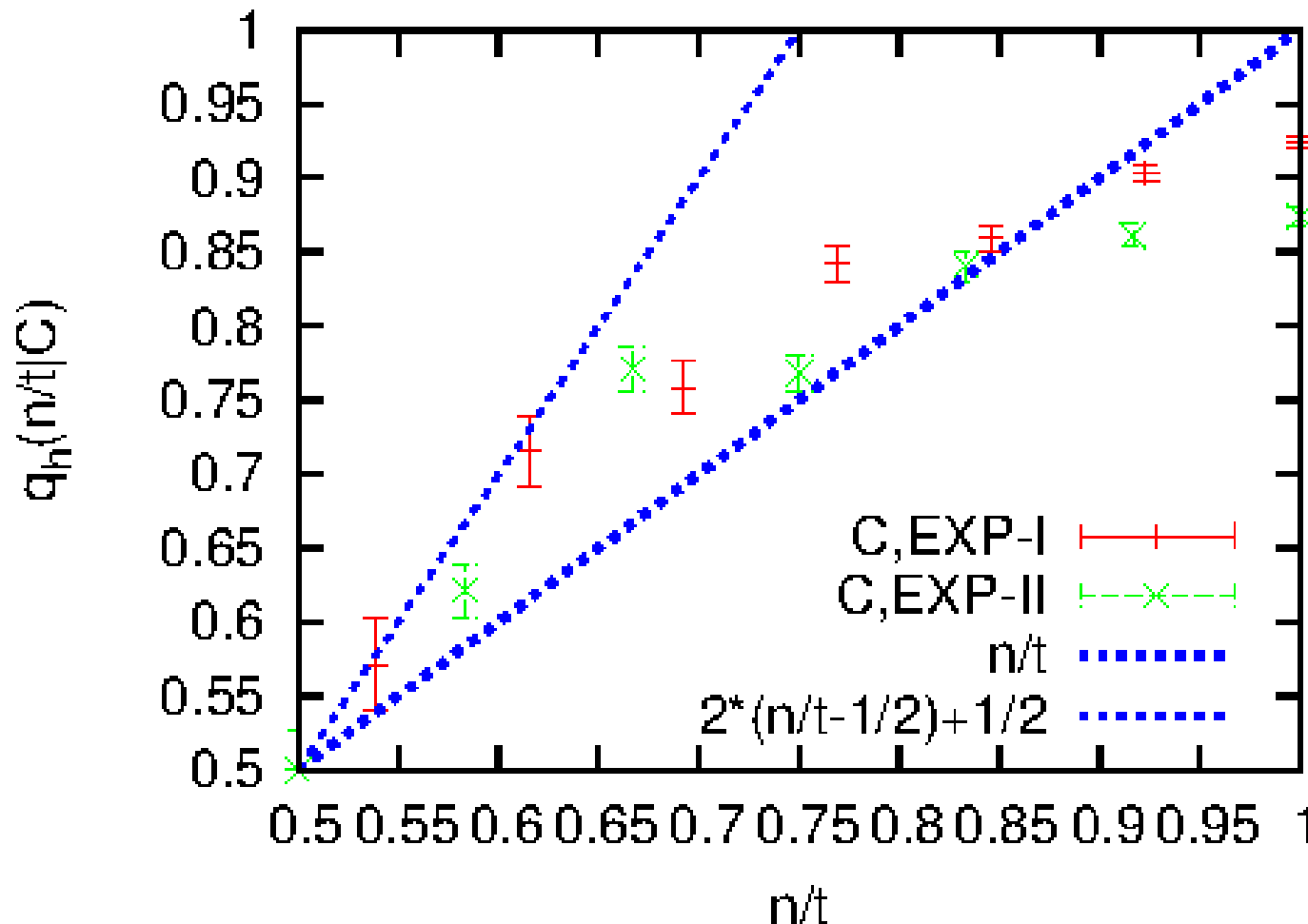
?



# Herder's Response Function $q_h(n/t|r)$

$$q_h(n/t|r) = 1 - q_h(1 - n/t|r)$$

## Case C

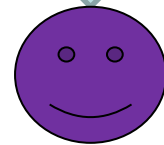


Case M



$$\{C_A(t), C_B(t)\}$$

**Zero-sum game**

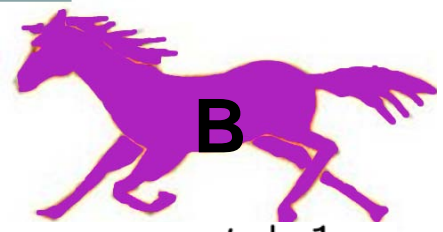


$$q_h(C_A(t)/t|M)$$

$$q_h(C_B(t)/t|M)$$

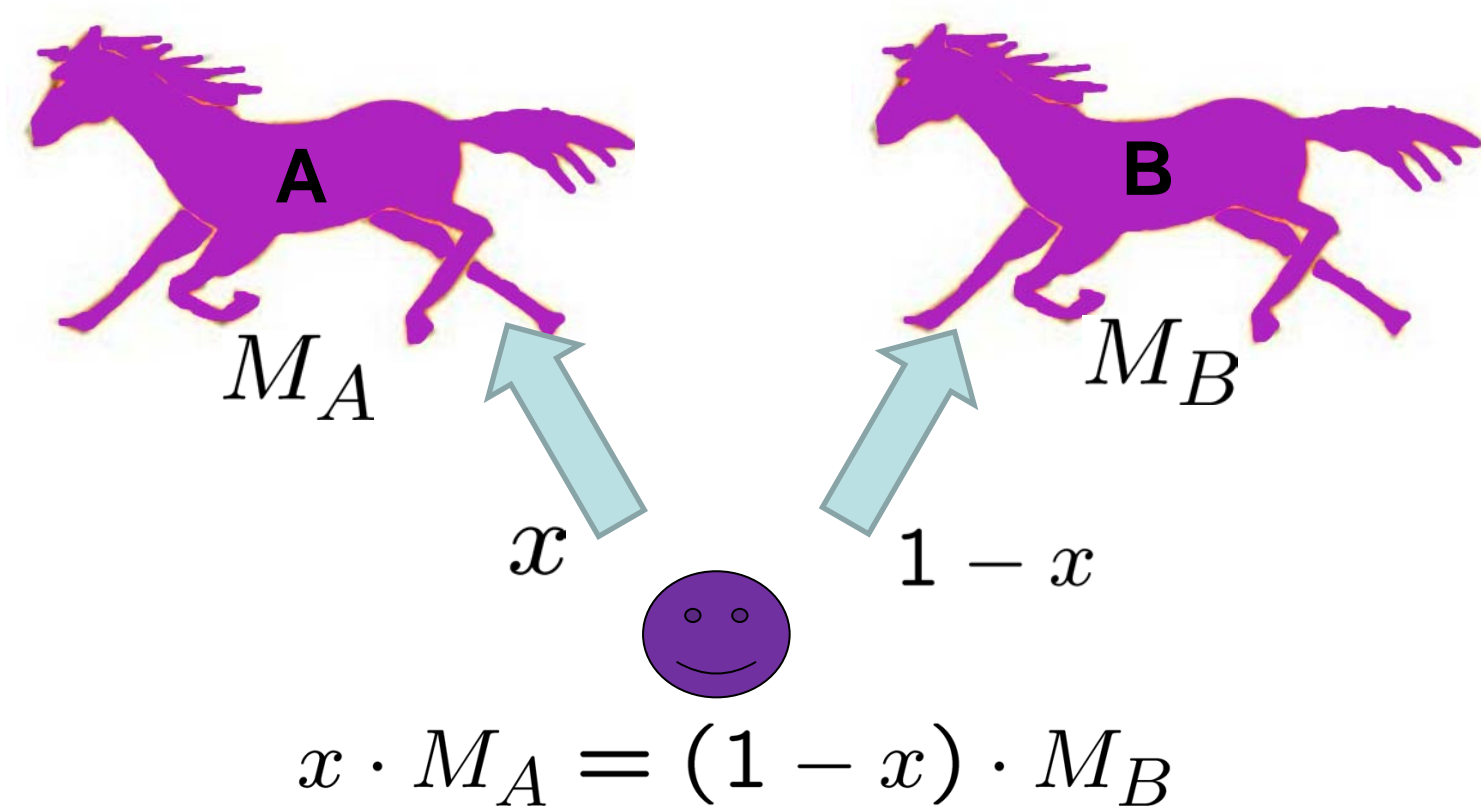


$$M_A = \frac{t+1}{C_A(t)+1}$$



$$M_B = \frac{t+1}{C_B(t)+1}$$

# Optimal Strategy = Max-Min Strategy

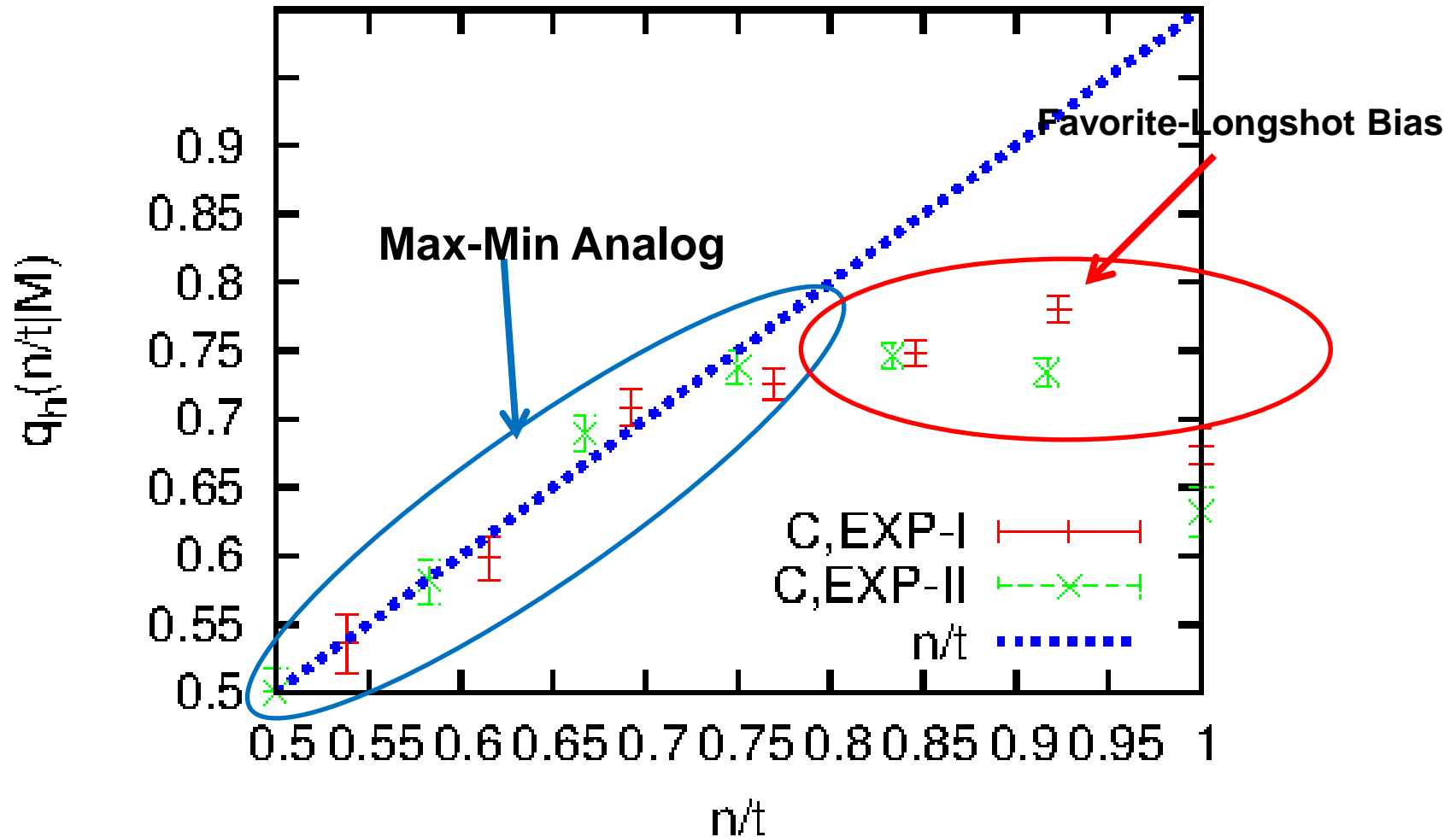


$$x \propto 1/M_A \propto C_A \longrightarrow q_h(n/t|M) = n/t$$

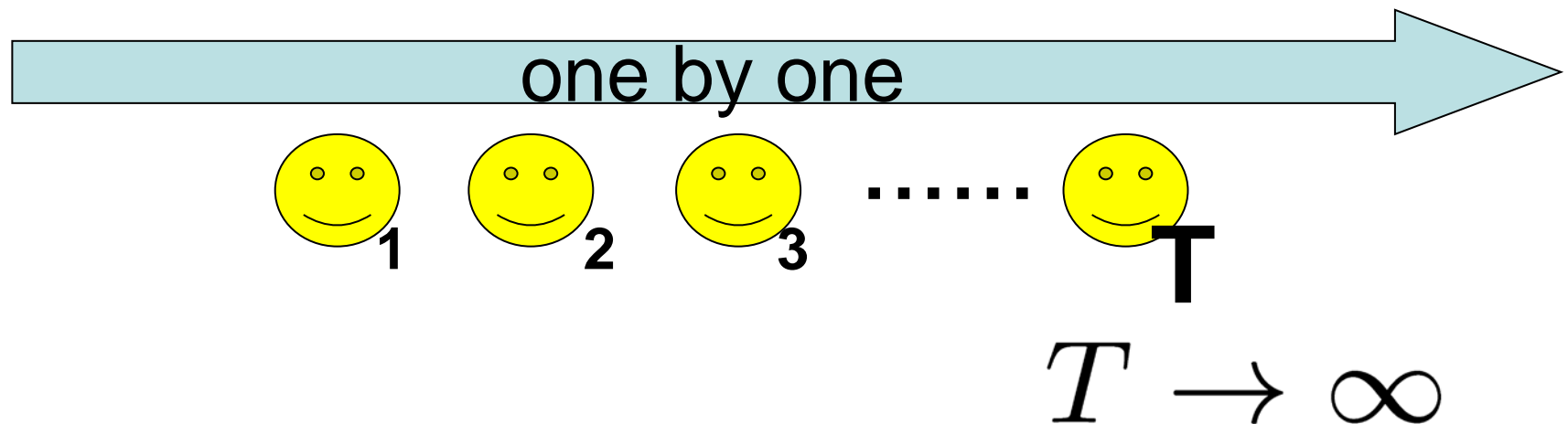
**Analog Herder**

# Herder's Response Function $q_h(n/t|r)$

## Case M



Q. What happens  
at the macroscopic level ?



$$z = \lim_{T \rightarrow \infty} Z(T|r), r \in \{C, M\}$$

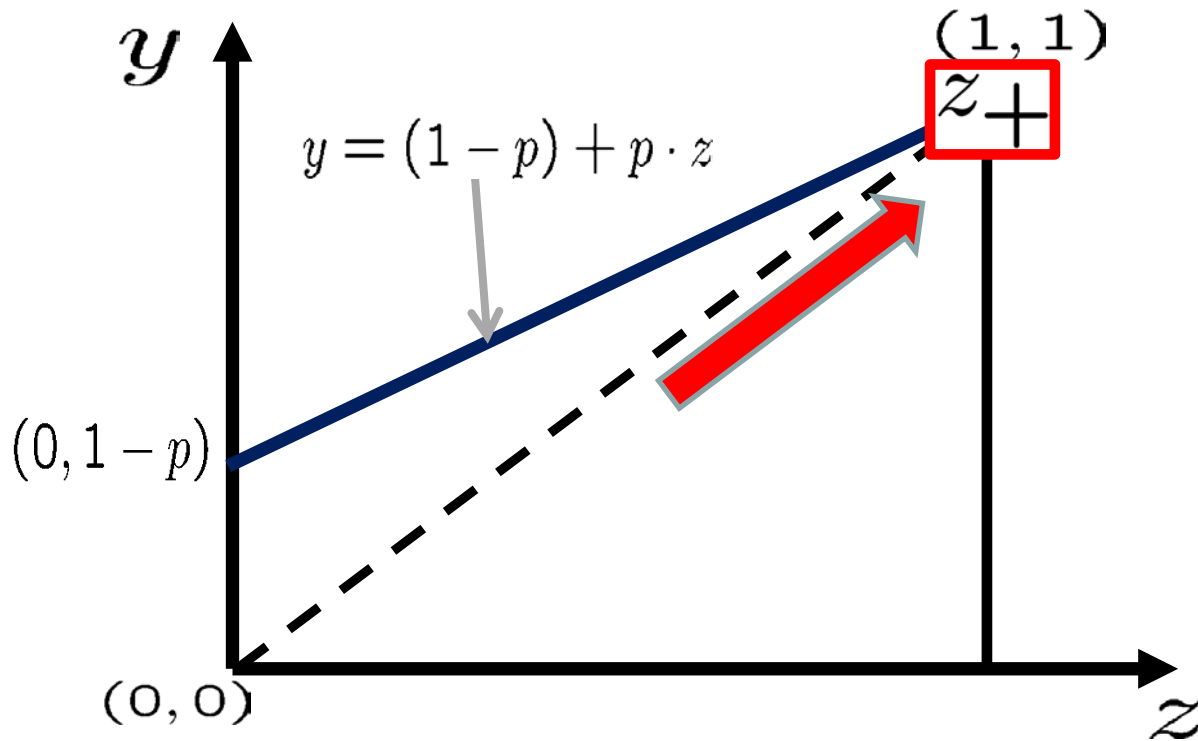


# Self-Consistent equation

$$z = (1 - p) \cdot 1 + p \cdot q_h(z|r) = q(z|r)$$



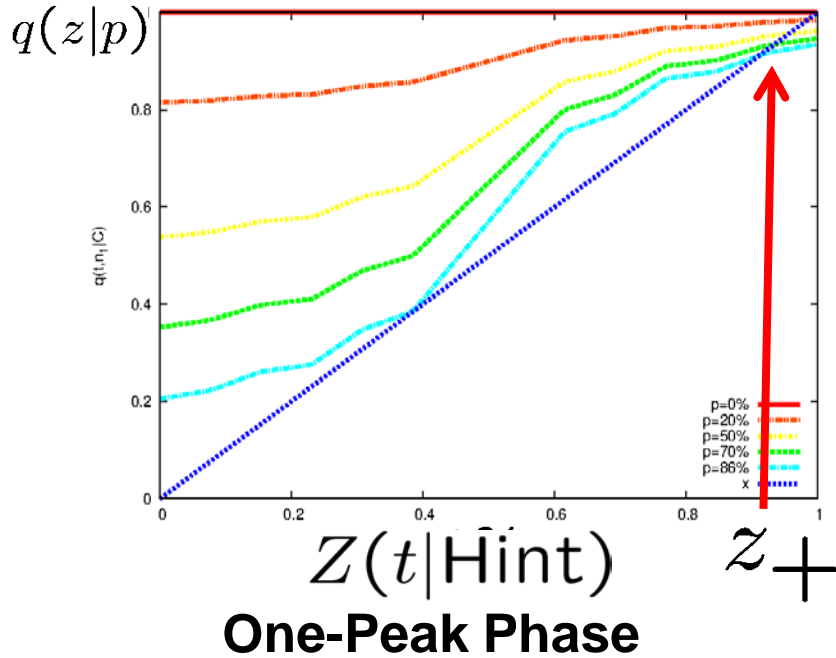
**Analog Herder :**  $q_h(z) = z$



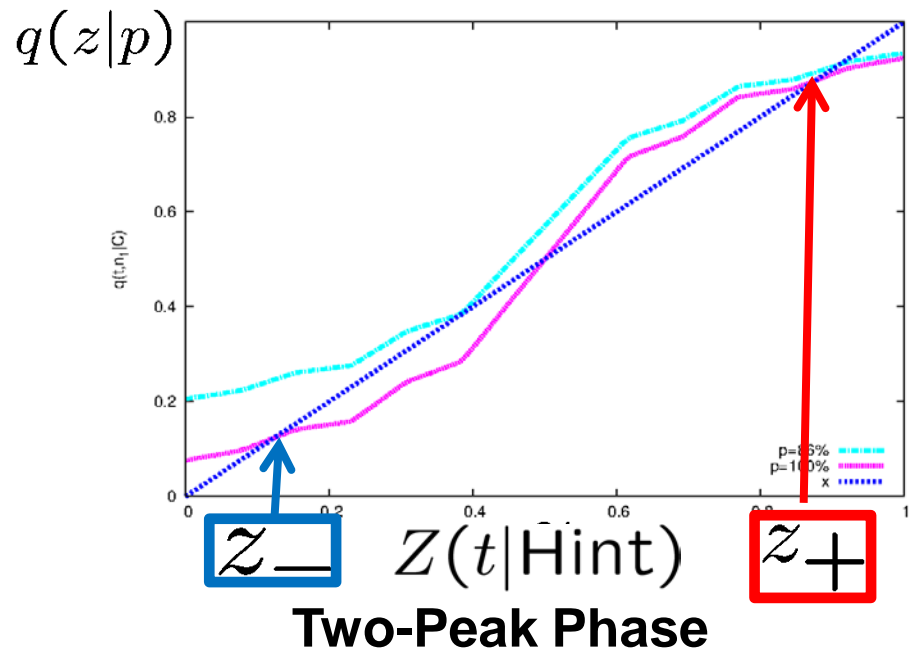
$$z_+ = 1 \text{ for } p < 1$$

# Case C

$$p \leq p_c(C) = 86\%$$



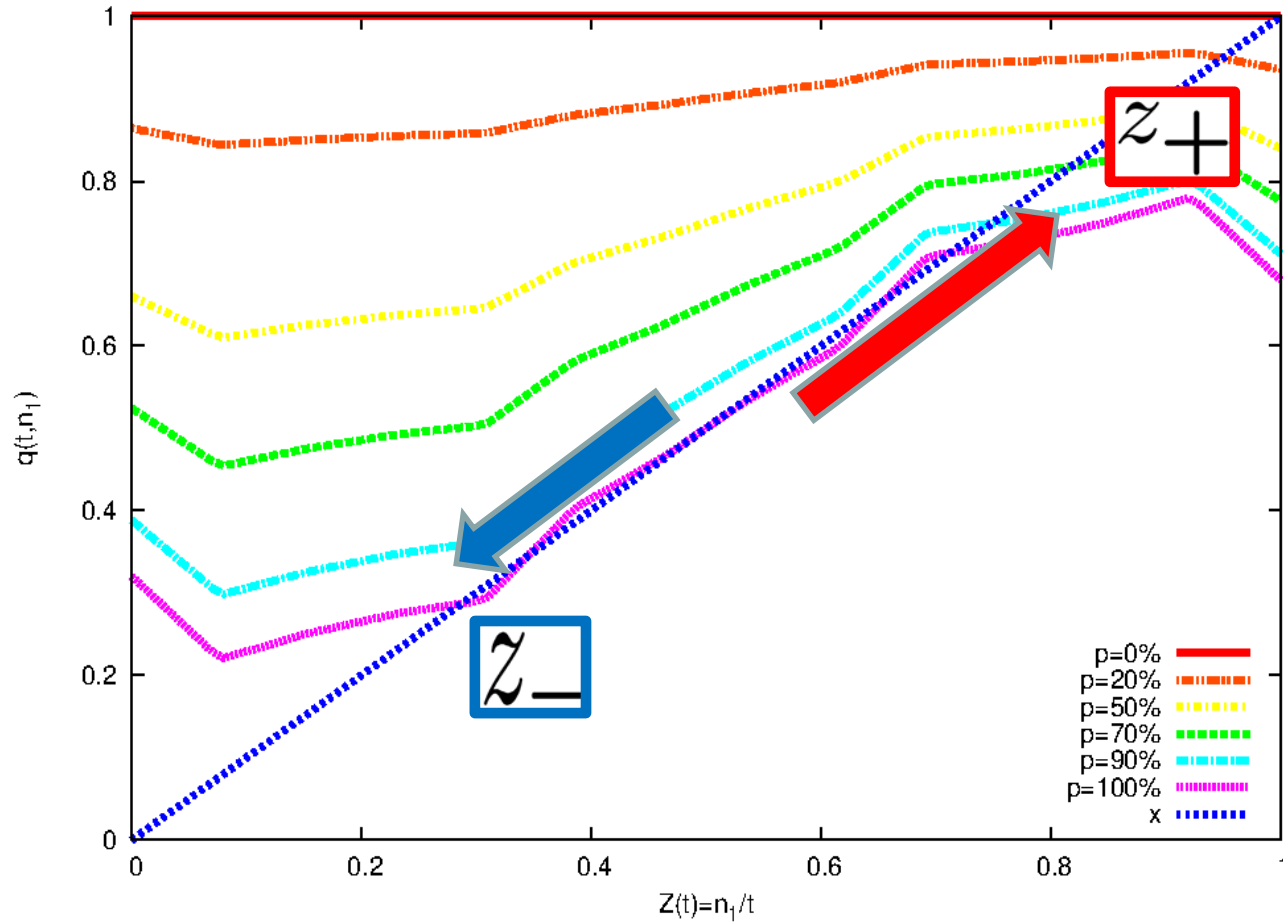
$$p \geq p_c(C) = 86\%$$



# Information Cascade Phase transition

S.Mori, M. Hisakado and T. Takahashi, Phys.Rev.E(2012).

# Case M



$$p_c(M) = 96\%$$

# Summary

## Microscopic Level

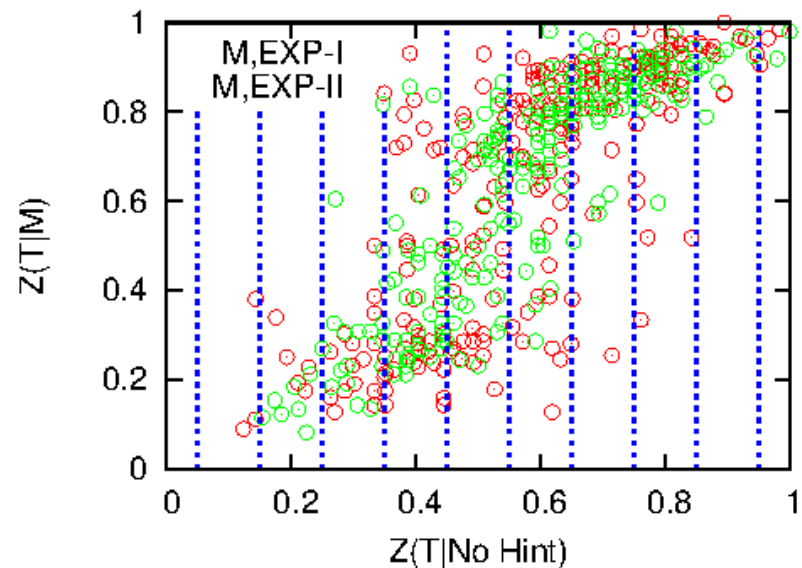
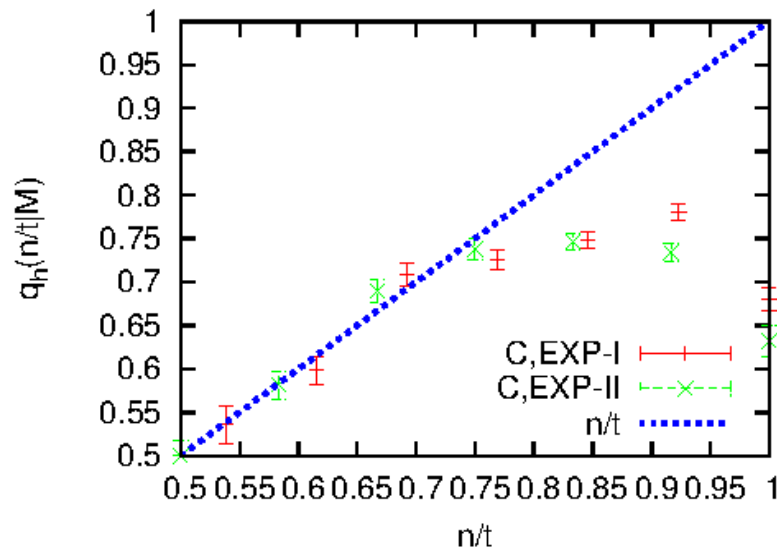
Max-Min Strategy is optimal.  $q_h(n/t|M) = n/t$

Herders collectively adopt it for  $4/3 < m < 4$  (or  $1/4 < n/t < 3/4$ ).

## Macroscopic Level

Max-Min Strategy is “optimal”.  $\lim_{T \rightarrow \infty} Z(T) = 1$  for  $p < 1$

Herder's % of Correct Choice is not so high by the bias.

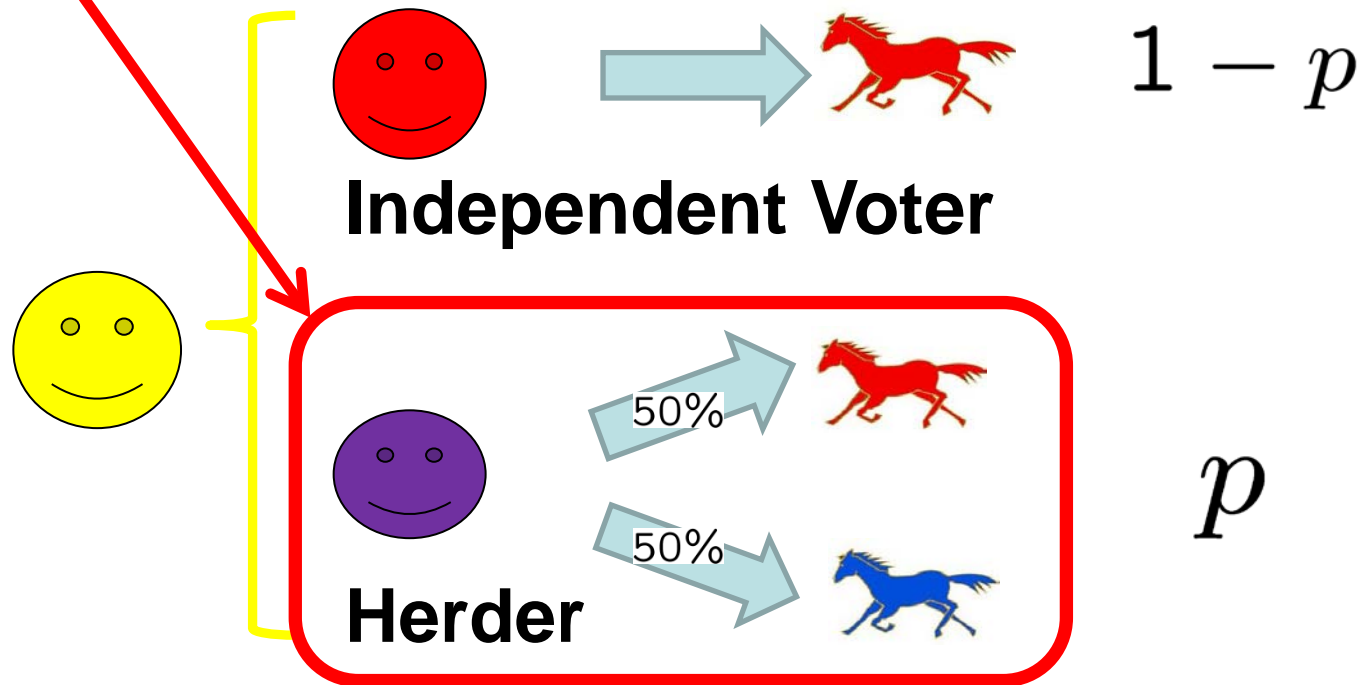


## Reference

Collective Adoption of Max-Min Strategy in an information cascade voting experiment  
S.Mori, M. Hisakado and T. Takahashi, J.Phys.Soc.Jpn.82(2013)084004.

**Thank you**

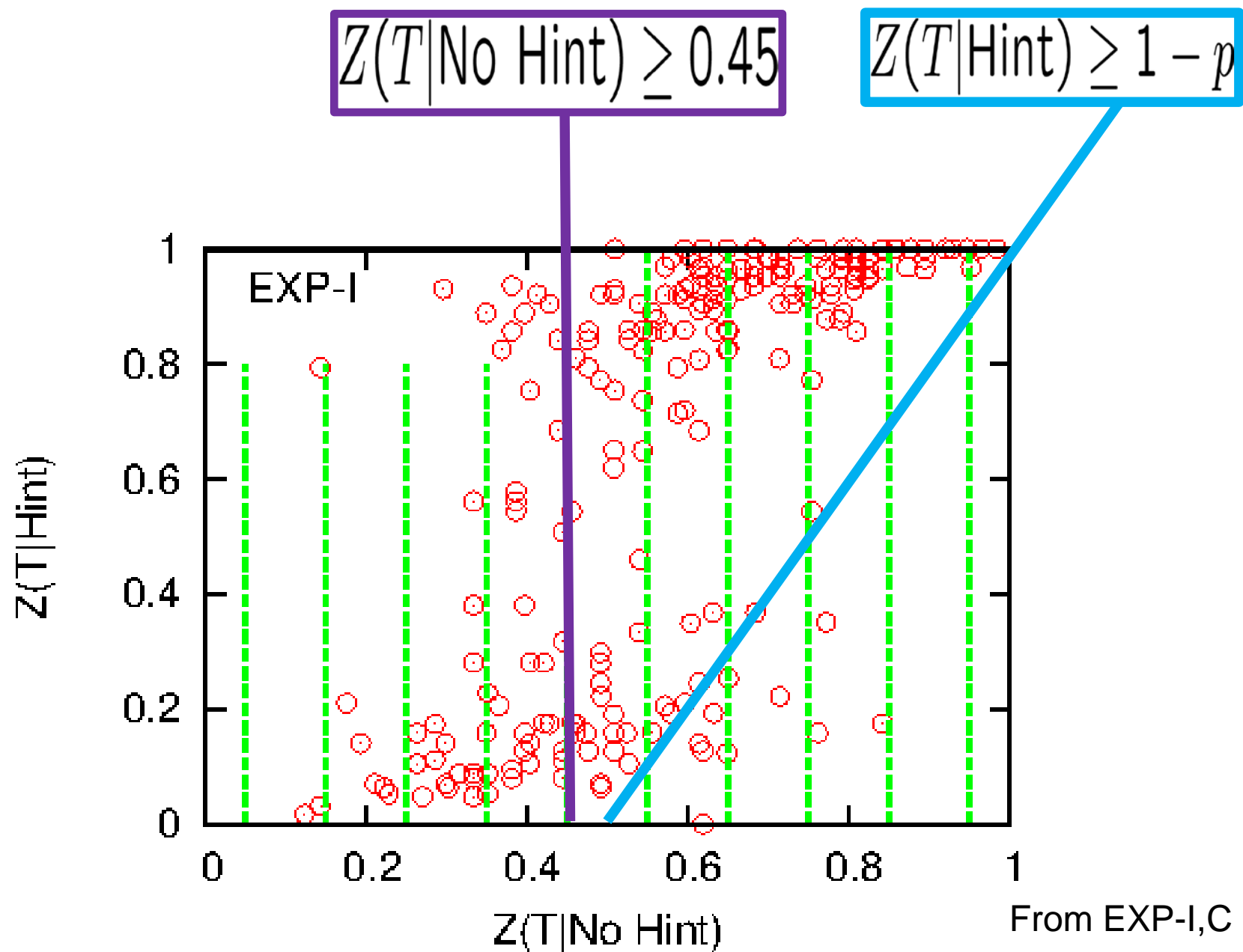
# Precondition of Experiment = No Bias



$$Z(T|\text{No Hint}) = (1 - p) \cdot 1 + p \cdot \frac{1}{2} = 1 - \frac{1}{2}p$$

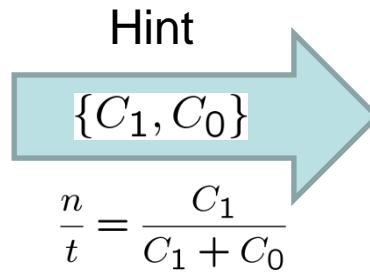
$$\longrightarrow Z(T|\text{No Hint}) \geq \frac{1}{2}, \quad Z(T|\text{Hint}) \geq 1 - p$$

We eliminate data which does not satisfy these conditions.



How to get  $q_h(n/t)$

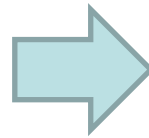
	# of Choices
Correct	$N_1(\text{No Hint})$
Wrong	$N_0(\text{No Hint})$
Total	$N$



	# of Choices
Correct	$N_1(\text{Hint})$
Wrong	$N_0(\text{Hint})$
Total	$N$

$$\frac{N_1(\text{No Hint})}{N} = (1 - p) \cdot 1 + p \cdot \frac{1}{2}$$

Estimate of  $p$

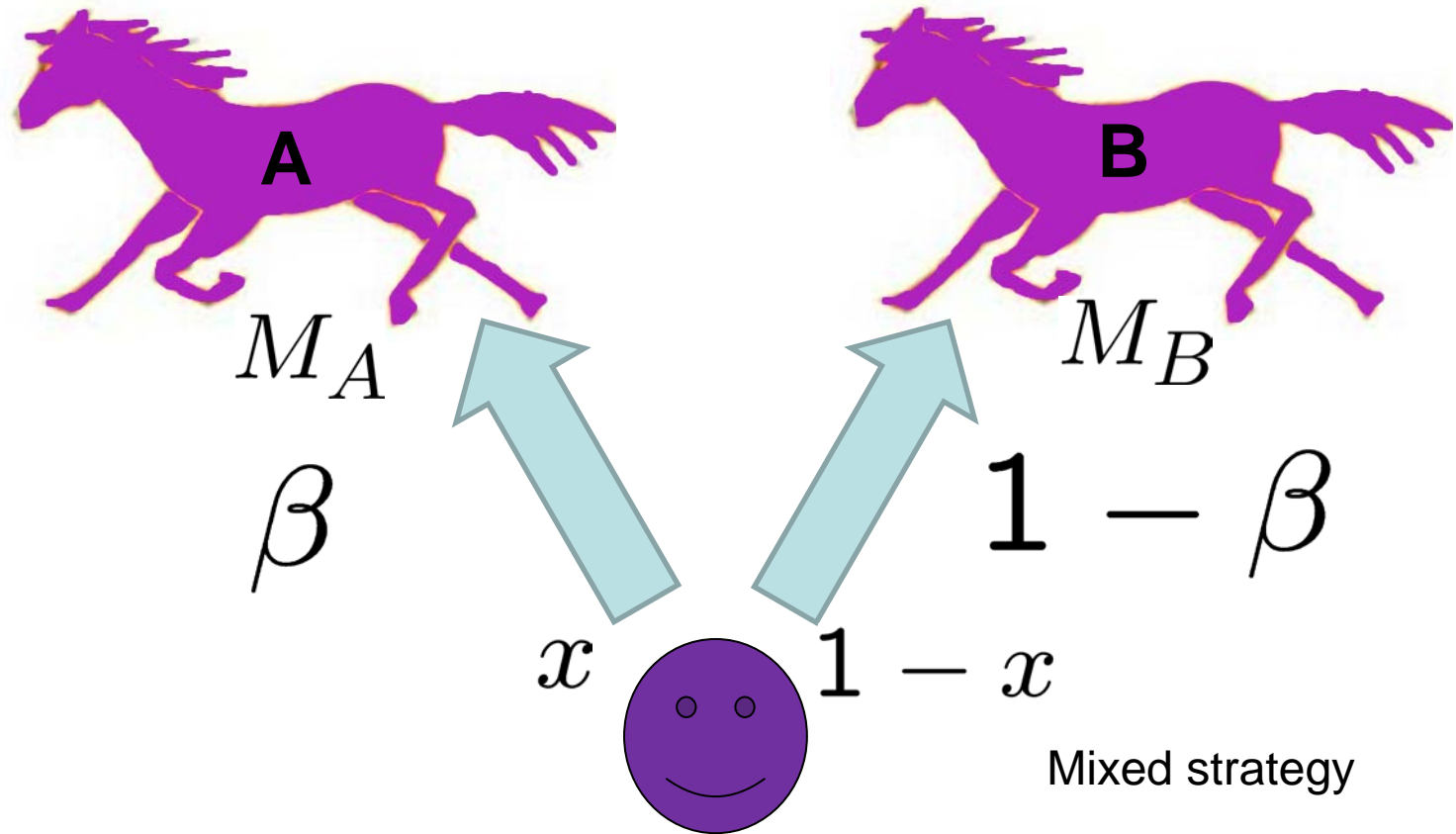


$$\frac{N_1(\text{Hint})}{N} = (1 - p) \cdot 1 + p \cdot q_h\left(\frac{C_1}{C_1 + C_0}\right)$$

Estimate of  $q_h$



# Optimal Strategy = Maximization of Expected Return



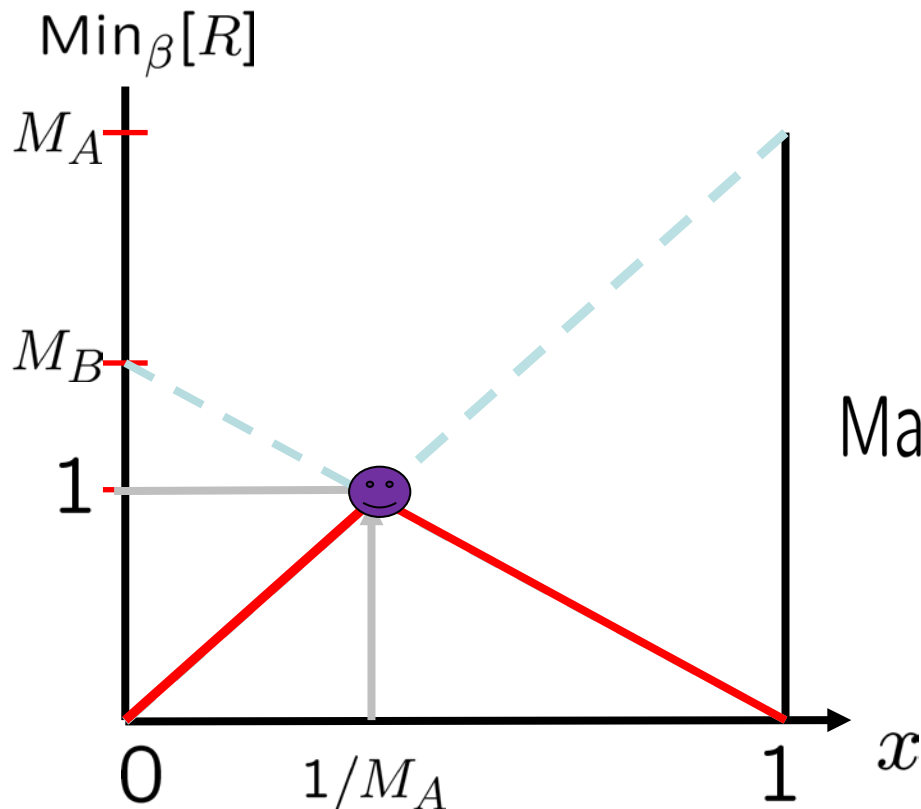
Expected Return

$$R = x \cdot \beta \cdot M_A + (1 - x) \cdot (1 - \beta) \cdot M_B$$

# Optimal Strategy=Maximization of Expected Return

$$\begin{aligned} R &= x \cdot \beta \cdot M_A + (1 - x) \cdot (1 - \beta) \cdot M_B \\ &= \beta(x \cdot M_A - (1 - x) \cdot M_B) + (1 - x) \cdot M_B \end{aligned}$$

# Max-Min Strategy=Maximization of Minimum value of R



$$\text{Max}_x(\text{Min}_\beta[R])_x = 1 \text{ at } x = 1/M_A$$