

## 情報カスケードにおけるドミノ効果のスケーリング理論

**S.Mori    Kitasato University**

- (1)情報カスケードとドミノ効果
- (2)相関関数のスケール則
- (3)実験と解析結果
- (4)まとめ

### Collaborators

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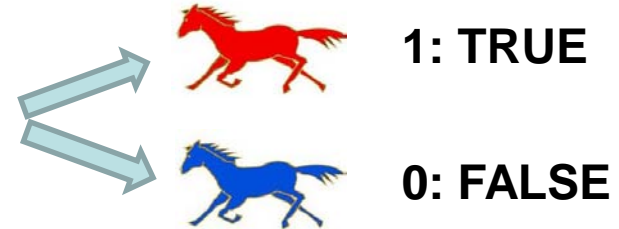
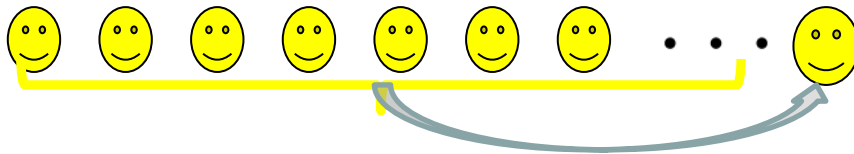
# Information cascade

- (1) X is A or B ? : Discrete Choice.
- (2) Sequential choices, one by one.
- (3) Private signal (secret).
- (4) One watches others' choices .



Choose the majority  
irrespective of one's private signal  
= Information cascade

Sequential Voting with Two-choice quiz (EXP-I)



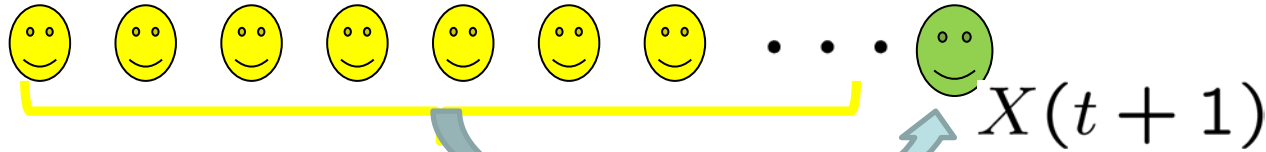
$t$ : order	1	2	3	4	5	6
Private Signal ( in mind)	a	a	b	b	b	b
Choice	A	A	A	A	A	A



Information Cascade Starts

## 2 questions

1. How others' choice affect one's choice ?  $X(t) \in \{1 : \text{True}, 0 : \text{False}\}$



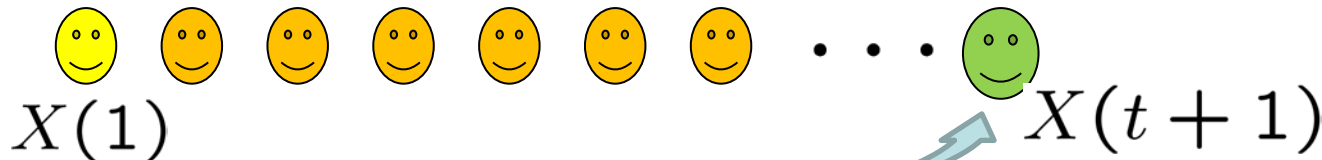
$z(t)$  Ratio of correct choices of first  $t$  subjects

$$Z(t) = \sum_{s=1}^t X(s)/t$$

$$q(z) = \Pr(X(t+1) = 1 | z(t) = z)$$

**Response function**

2. How 1<sup>st</sup> subject's choice affect one's choice ?



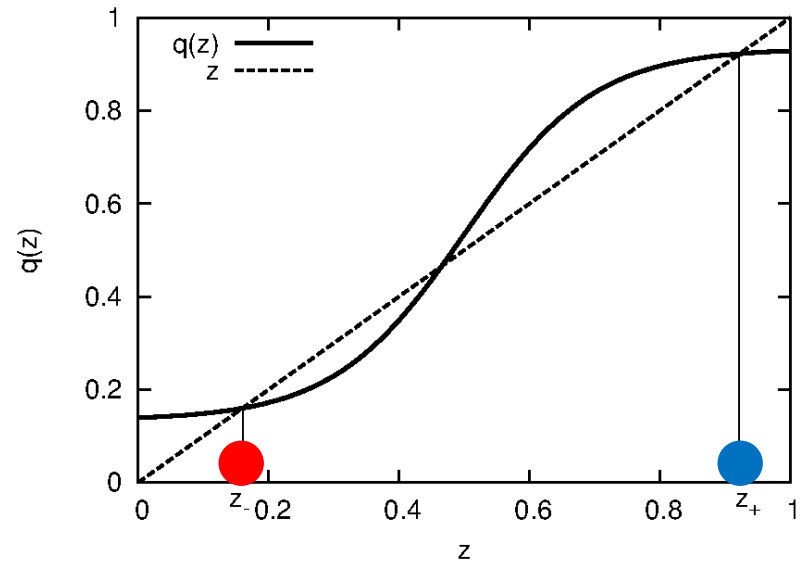
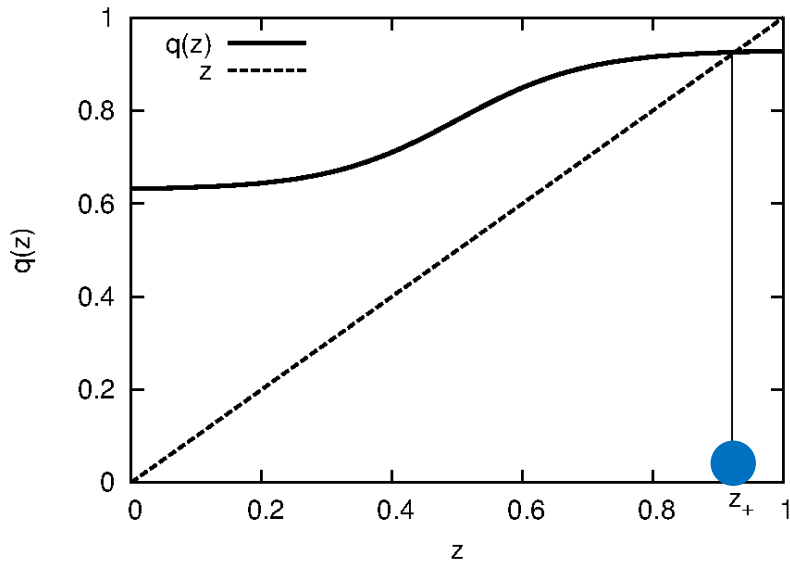
**Information Transmission**

$$C(t) = \text{Cov}(X(1), X(t+1))$$

$$C(t)/C(0) = \Pr(X(t+1) = 0 | X(1) = 0) - \Pr(X(t+1) = 0 | X(1) = 1)$$

Measure of Domino Effect





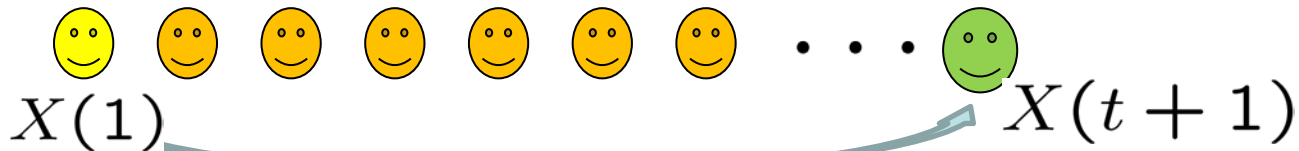
**One-Peak Phase**  $\lim_{t \rightarrow \infty} z(t) = z_+$

**Two-Peaks Phase**  $\lim_{t \rightarrow \infty} z(t) = z_+ \text{ or } z_-$

S.Mori and M.Hisakado(2014)

**Order parameter =Domino Effect lasts forever or not ?**

$$C(t)/C(0) = \Pr(X(t+1) = 0|X(1) = 0) - \Pr(X(t+1) = 0|X(1) = 1)$$



$$C(t)/C(0) \simeq a \cdot t^{l-1} \rightarrow 0, l < 1$$

**One-Peak Phase**

$$C(t)/C(0) \simeq c + a \cdot t^{l-1} \rightarrow c > 0$$

**Two-Peaks Phase**

c: order parameter

# How to estimate c: order parameter

S.Mori , M.Hino, M.Hisakado and T.Takahashi(2014)

$$\xi(t) = \sqrt{\frac{\sum_{s=0}^{t-1} C(s)s^2}{\sum_{s=0}^{t-1} C(s)}}$$

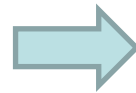
2<sup>nd</sup> moment correlation time

$$\tau(t) = \sum_{s=0}^{t-1} C(s)/C(0)$$

Integrated correlation time

## One-Peak Phase

$$C(t)/C(0) \propto t^{l-1} \rightarrow 0$$

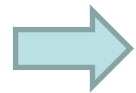


$$\xi(t)/t \rightarrow \sqrt{l/(l+2)} < 1/\sqrt{3}$$

$$\tau(t)/t \propto t^{l-1} \rightarrow 0$$

## Two-Peaks Phase

$$C(t)/C(0) \rightarrow c > 0$$

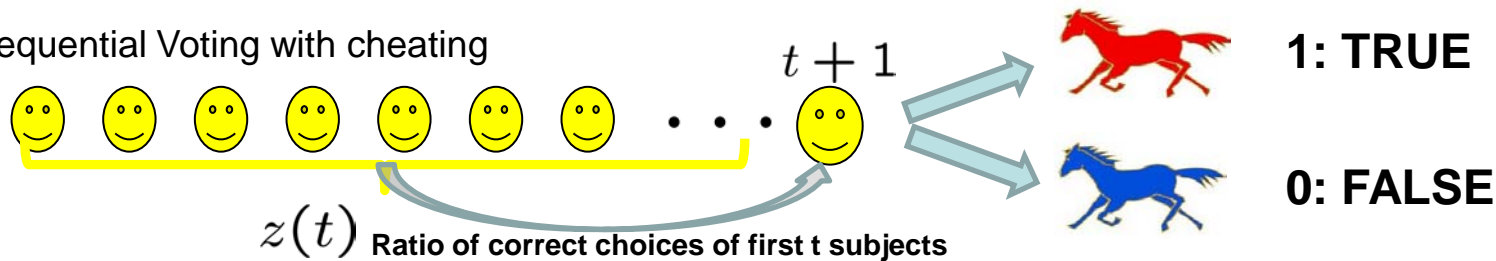


$$\xi(t)/t \rightarrow 1/\sqrt{3}$$

$$\tau(t)/t \rightarrow c$$

# Information Cascade Experiment I : Quiz

Sequential Voting with cheating



## EXP-I : Quiz

**Voting Experiment**

No. Info.

Q.30: Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky B : Beethoven

**Private signal**

Answer

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**Voting Experiment**

All previous subjects' Info.

Up to now 7 subjects have answered.

Their choices are as follows. Please choose.

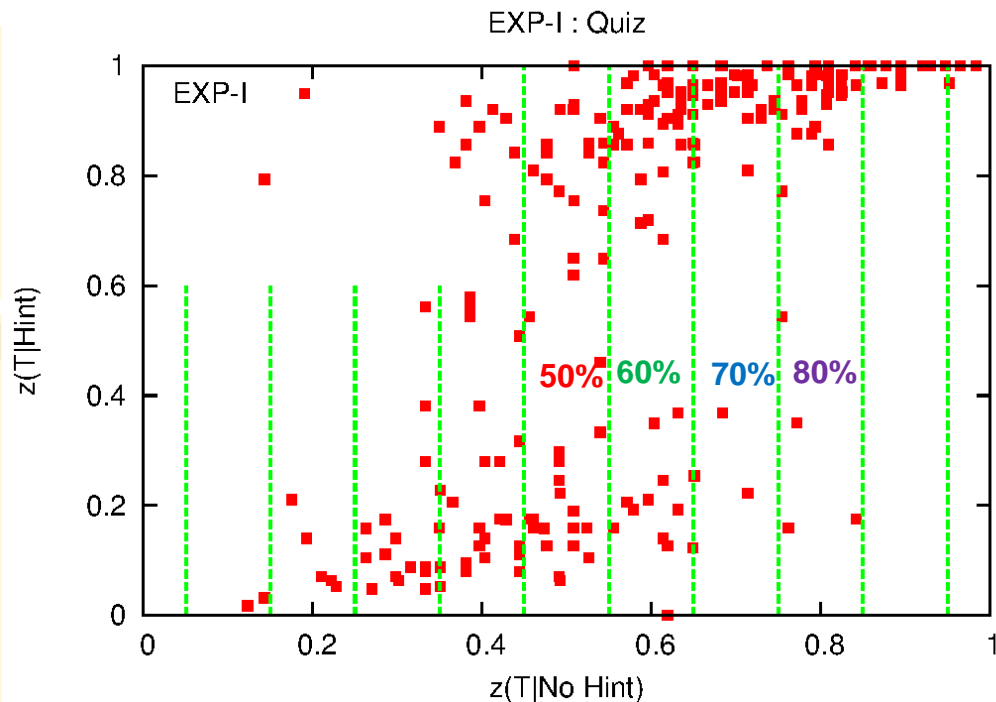
Q.30: Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky B : Beethoven

1 6

**Others' choices**

Answer



Experiment	T (Length)	# of Sequences
EXP-I (Quiz)	60	38(50%) + 52(60%) + 38(70%) + 38(80%) + 14(>85%) = 170

Classification by Private Signal

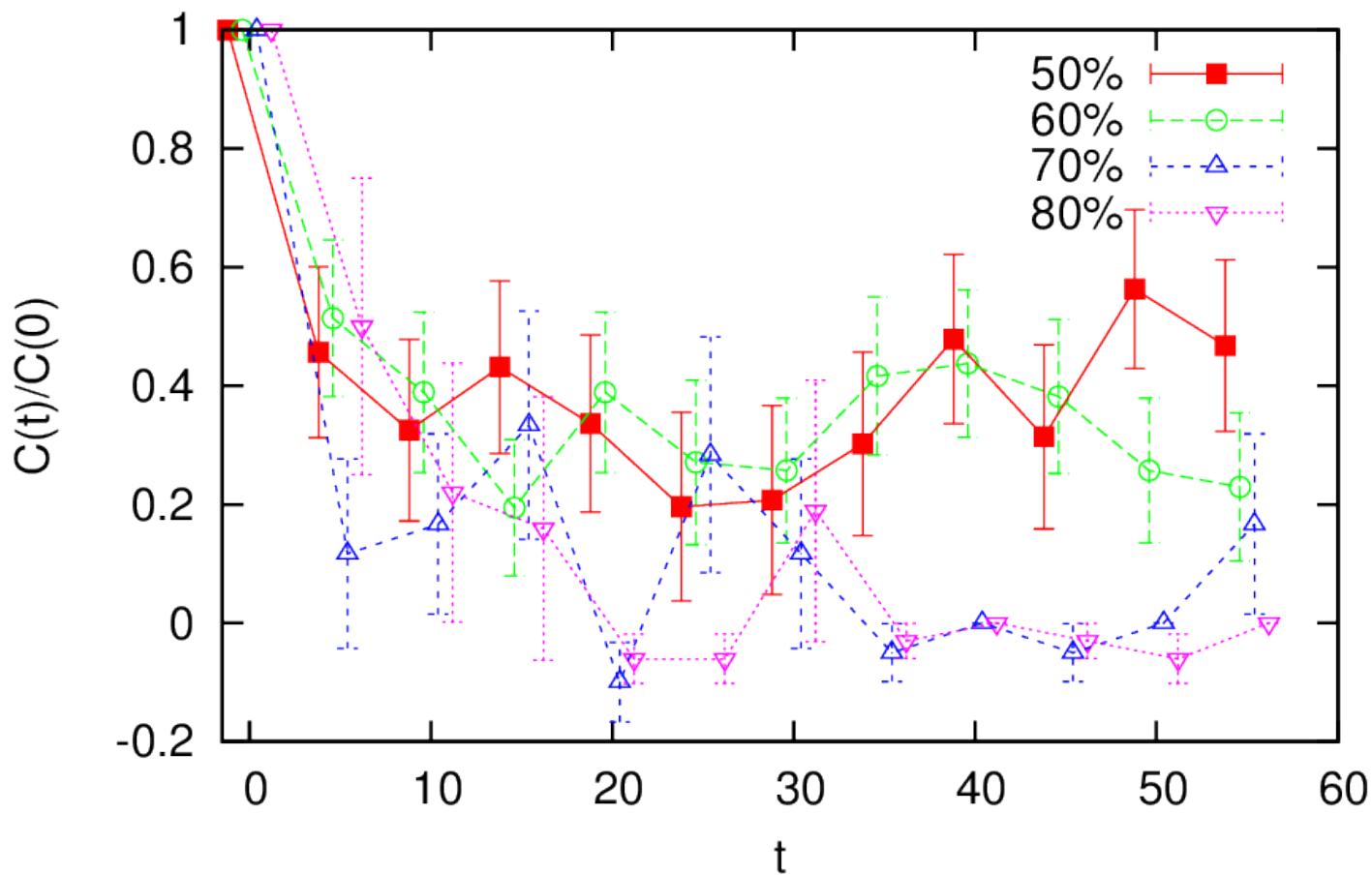
# Results in EXP-I

# Domino Effect in EXP-I

$C(t)/C(0) \propto t^{l-1} \rightarrow 0, l < 1$   $\longleftrightarrow$  One-Peak Phase

$C(t)/C(0) \rightarrow c > 0$   $\longleftrightarrow$  Two-Peak Phase

EXP-I : Quiz



# Results in EXP-I

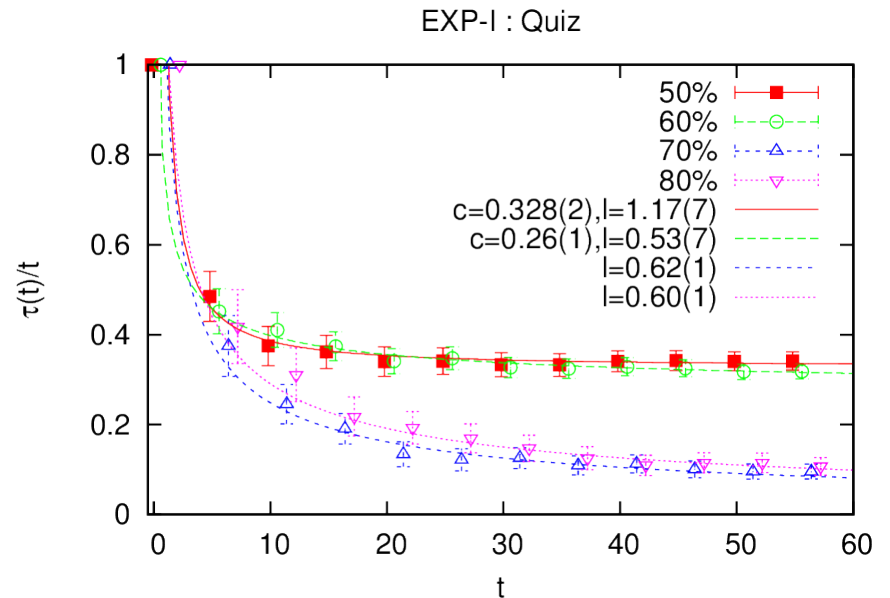
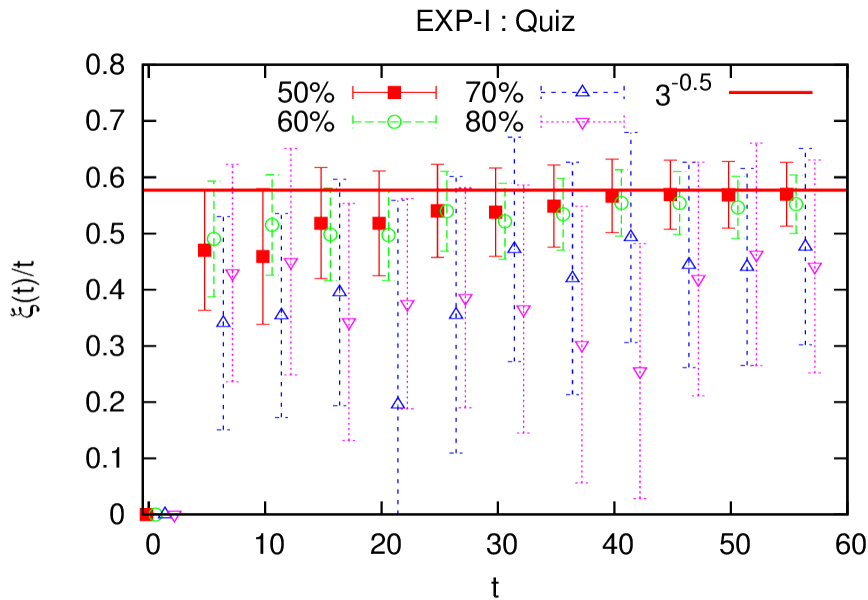
## Order parameter

One-Peak Phase

$$\left\{ \begin{array}{l} \xi(t)/t \rightarrow \sqrt{l/(l+2)} < 1/\sqrt{3} \\ \tau(t)/t \propto t^{l-1} \rightarrow 0 \end{array} \right.$$

Two-Peaks Phase

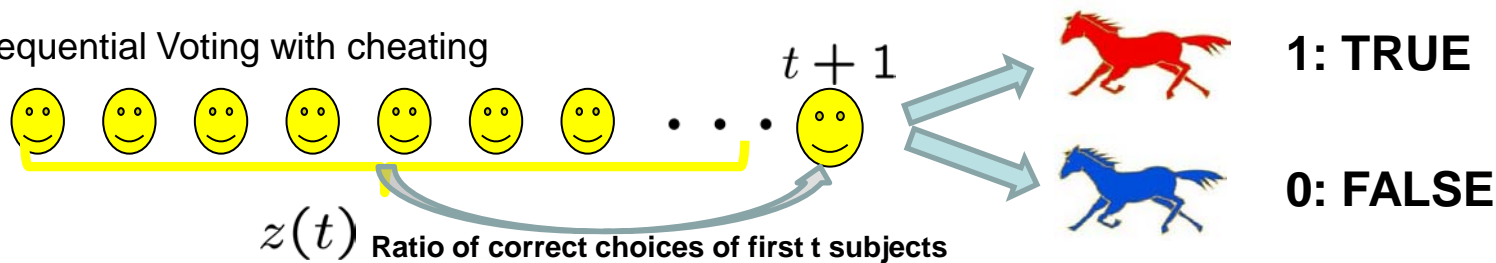
$$\left\{ \begin{array}{l} \xi(t)/t \rightarrow 1/\sqrt{3} \\ \tau(t)/t \rightarrow c \end{array} \right.$$



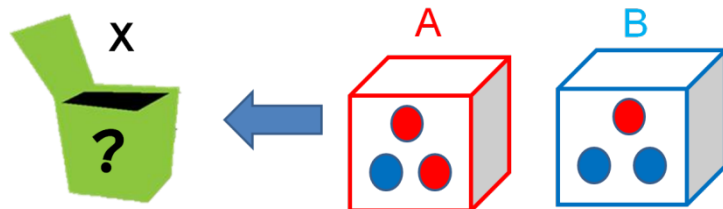


# Information Cascade Experiment II : Urn

Sequential Voting with cheating



EXP-II : Urn



**Cascade Experiment**

You have answered 5 questions up to now.

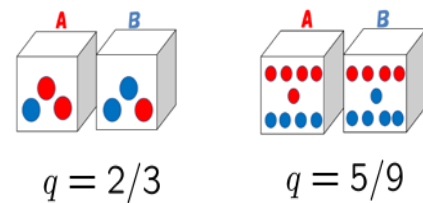
Your ball color is **RED**. ← **Private signal**

Which type, A or B ?

Please answer your confidence about your choice.

Urn	A					B					Urn		
votes	3					1					votes		
A	100%	90%	80%	70%	60%	50%	50%	60%	70%	80%	90%	100%	B
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	

Answer

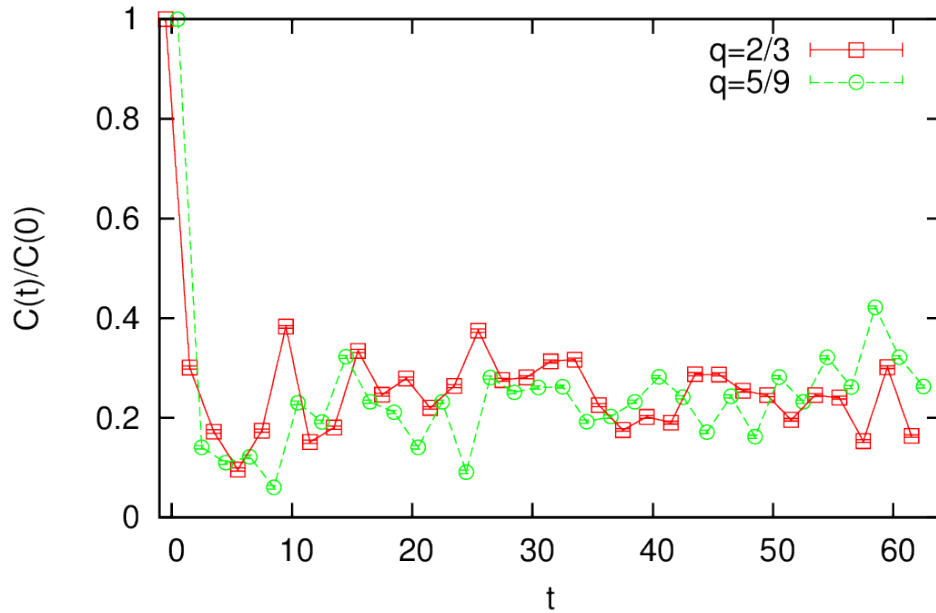


← **Others' choices**

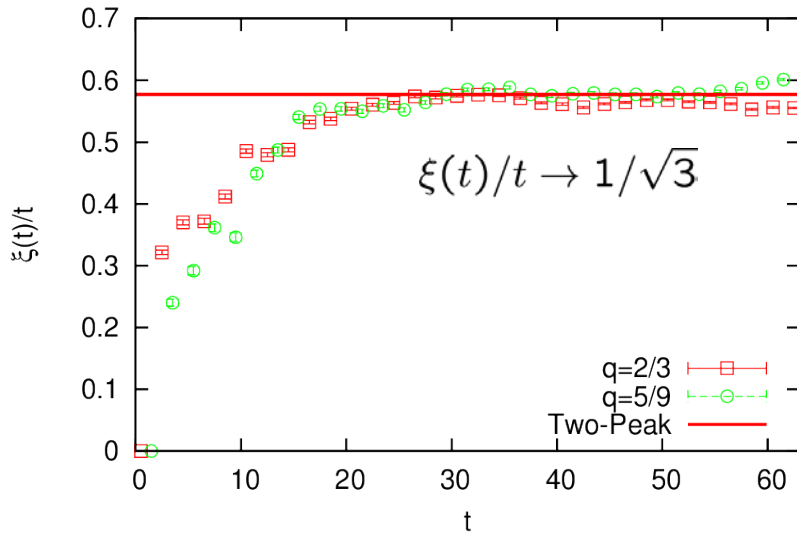
Experiment	T	# of Sequences
EXP-II (Urn)	63	200+200 ( $q=2/3, 5/9$ )

# Results in EXP-II

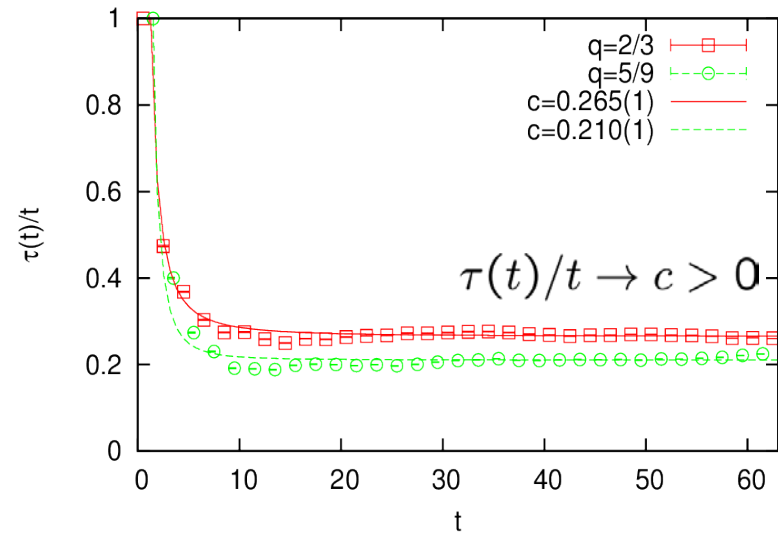
EXP-II : Urn



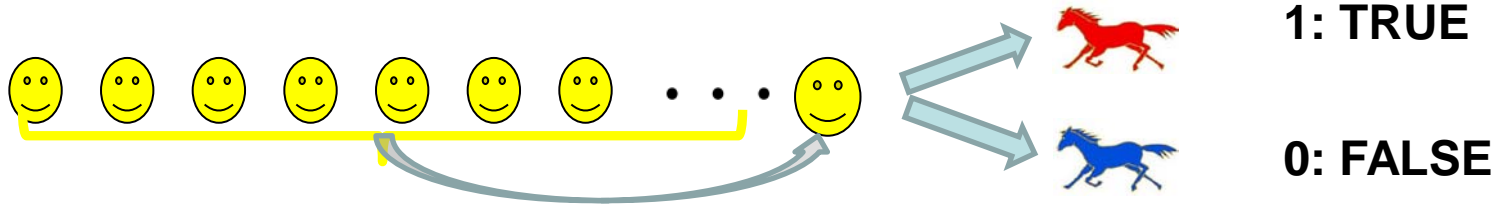
EXP-II : Urn



EXP-II : Urn



# Summary



$$C(t) = \text{Cov}(X(1), X(t + 1))$$

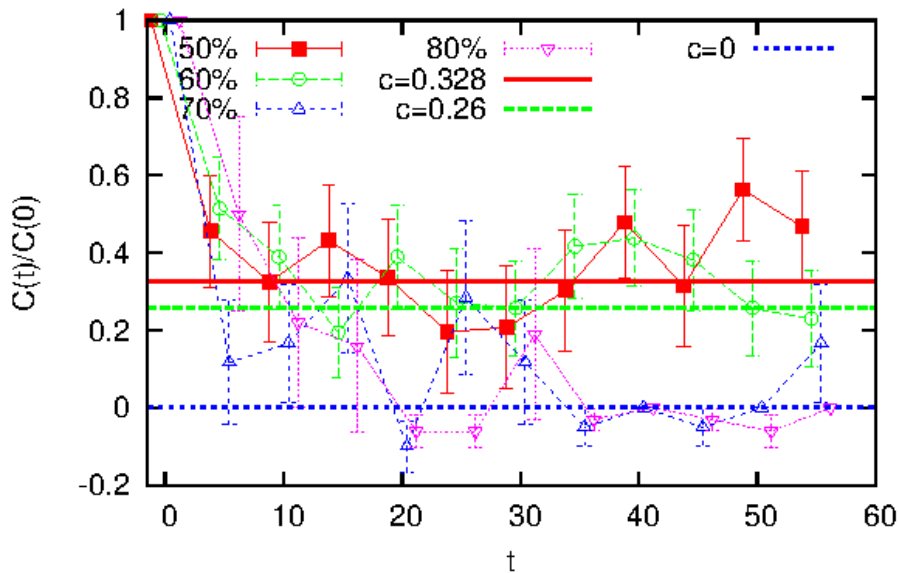
$$C(t)/C(0) \propto t^{l-1} \rightarrow 0, l < 1$$

$$C(t)/C(0) \rightarrow c > 0$$

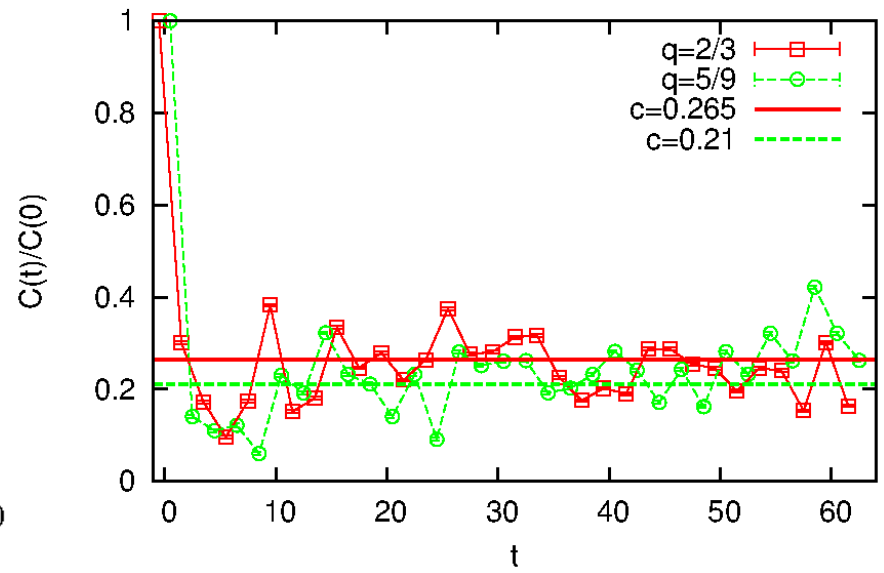
One-peak phase

Two-peaks phase

EXP-I : Quiz



EXP-II : Urn



# References

## Experiments

Phase transition to two-peaks phase in an information cascade voting experiment  
S.Mori, M. Hisakado and T. Takahashi, Phys.Rev.E86(2012)026109-026118.

Collective adoption of Max-Min strategy in an information cascade voting experiment  
S.Mori, M. Hisakado and T. Takahashi, J. Phys. Soc. Jpn. 82 (2013) 084004-084013

**Detection of non-self-correcting nature in information cascade**  
**S. Mori, M.Hino, M.Hisakado and T. Takahashi, submitted to the proceedings of ECCS'14.**

## Theoretical Studies

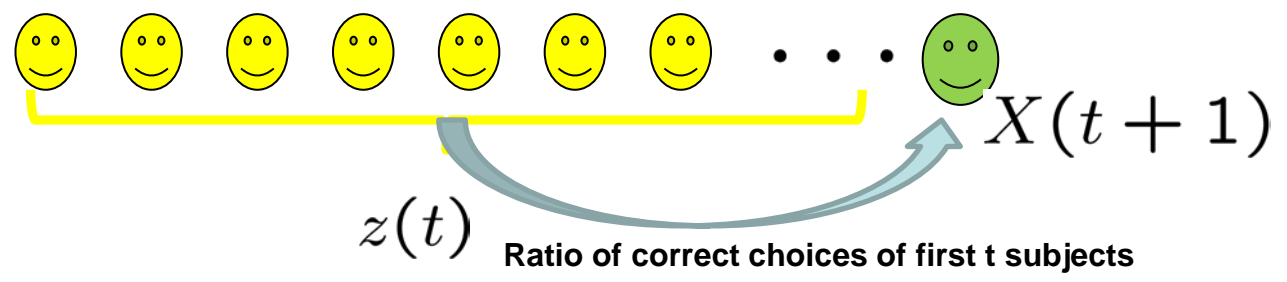
Digital Herders and Phase transition in a Voting model  
M. Hisakado and S.Mori, J.Phys.A44(2011)275204.

Finite-size scaling analysis of binary stochastic processes and universality classes of information cascade phase transition  
S.Mori and M. Hisakado, arXiv:1404.4921

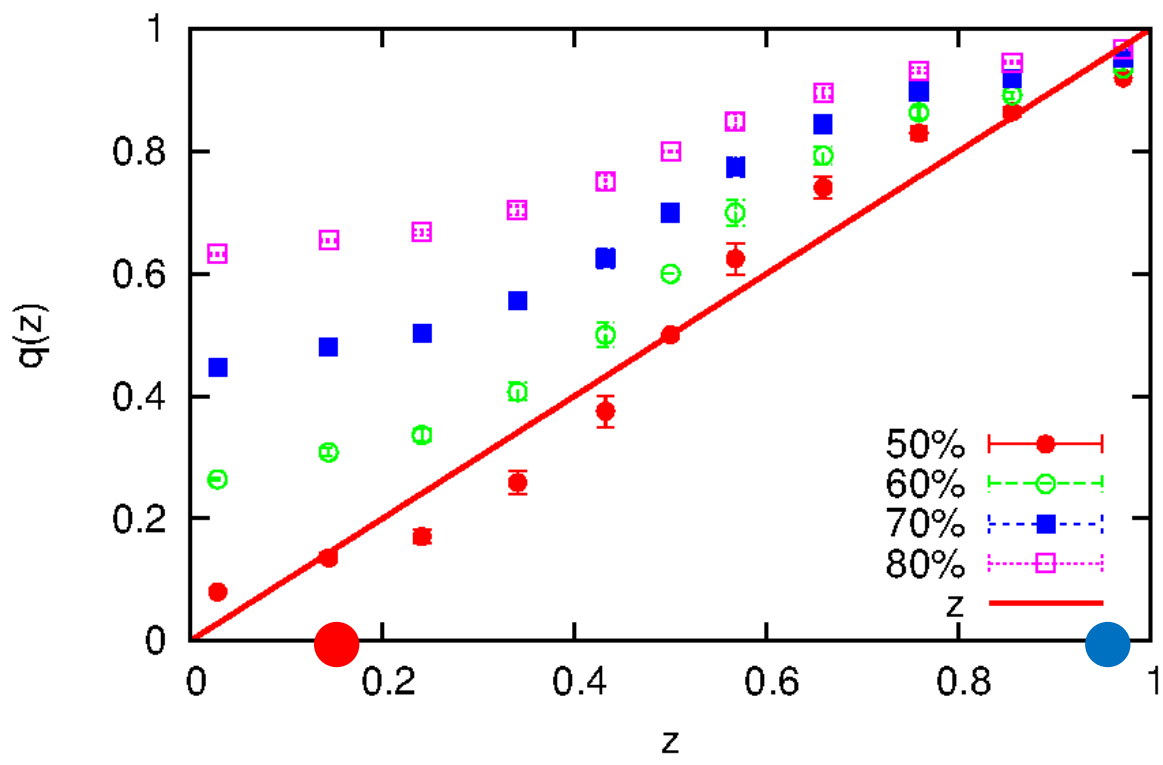
**Scaling analysis of critical properties for generalized polya urns**  
**S. Mori and M. Hisakado, in preparation.**

# Response function in EXP-I

$$q(z) = \Pr(X(t+1) = 1 | z(t) = z)$$

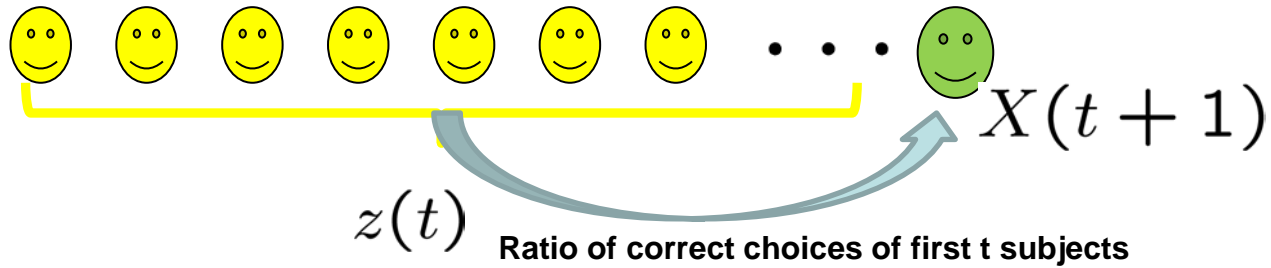


EXP-I : Quiz



# Response function in EXP-II

$$q(z) = \Pr(X(t+1) = 1 | z(t) = z)$$



EXP-II : Urn

