

Non-linear Pólya urn and Information Cascade

Human in Info. Cascade = Non-linear Pólya urn

universality class

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Corroborators

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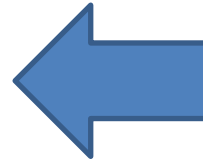
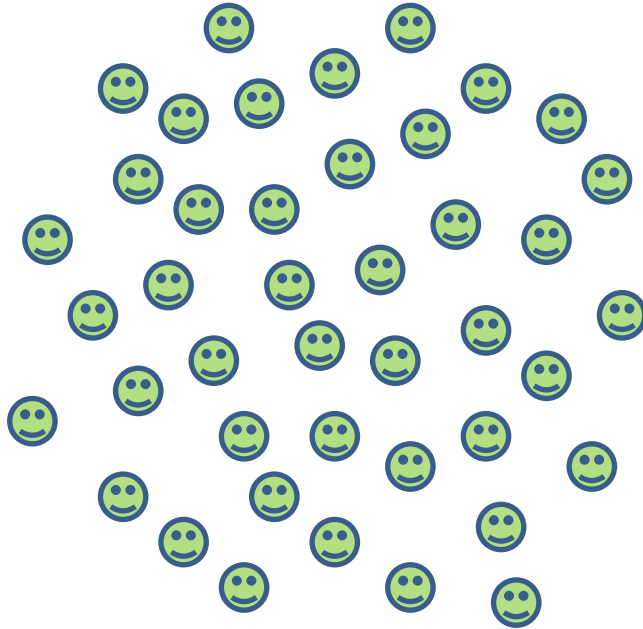
Masafumi Hino : NEC Corp.

Taiki Takahashi : Hokkaido Univ.

~~Model for Complex System~~

Effective Theory for Complex System

Macroscopic Phenomena



Effective Theory



Simple Model

Phase Transition

Universality class

- critical behaviors
- critical exponents
- scaling relation

Universality class

Magnet= Heisenberg Model

Ferromagnetic interaction

Fluid (Liquid-Gas)= Ising Model

Attractive interaction

Human= ?

Information cascade

Tendency to choose the majority
irrespective of one's private signal

Milosevic and H.Stanley 1976

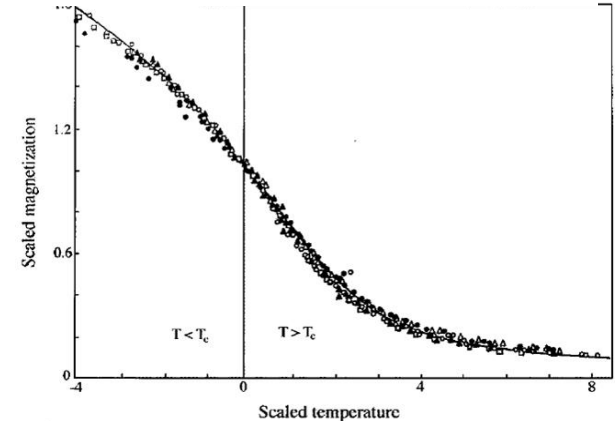
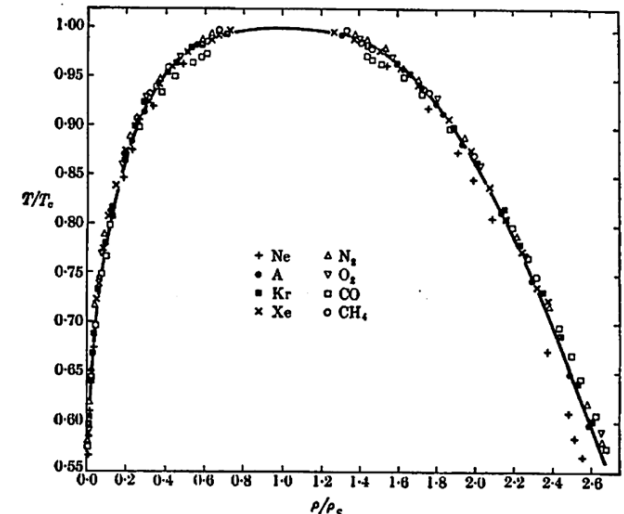


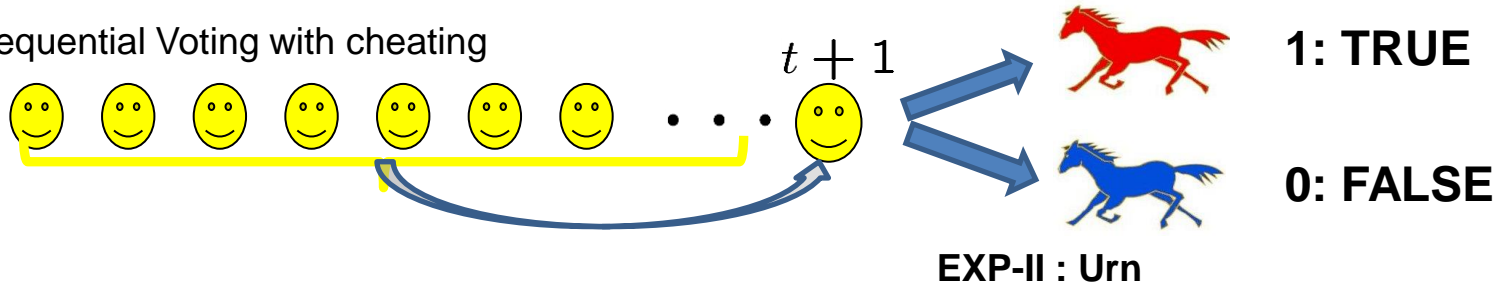
FIG. 1. Experimental *MHT* data on five different magnetic materials plotted in scaled form. The five materials are CrBr_3 , EuO , Ni , YIG , and Pd_3Fe . None of these materials is an ide-

H.Stanley Introduction to Phase Transitions
and Critical Phenomena, Oxford(1971)



Experimental Setup for Information cascade

Sequential Voting with cheating



Voting Experiment

No. Info.

Q.30: Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky B : Beethoven



Private signal

Answer

Voting Experiment

All previous subjects' Info.

Up to now 7 subjects have answered.

Their choices are as follows. Please choose.

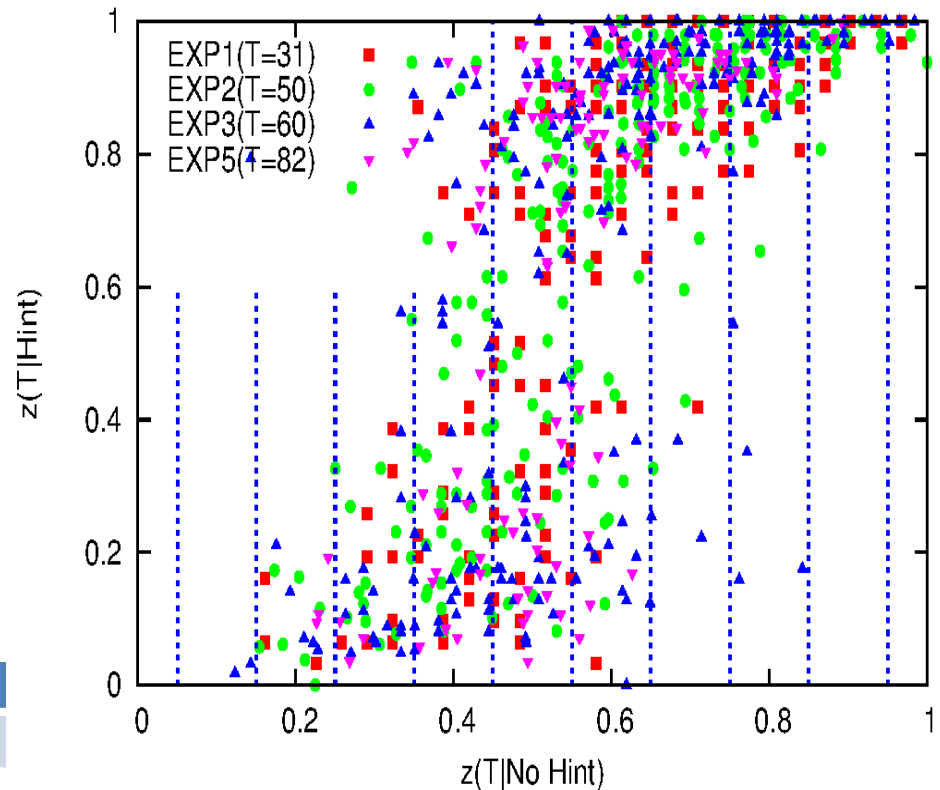
Q.30: Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky B : Beethoven



Social Info.

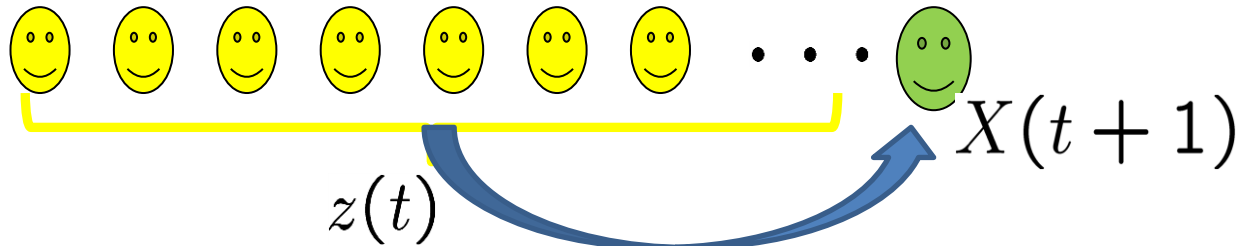
Answer



Exp.	Length T	# of Sequences
EXP3+5	60	73(50%)+79(60%)+46(70%)+43(80%)=241

Response function

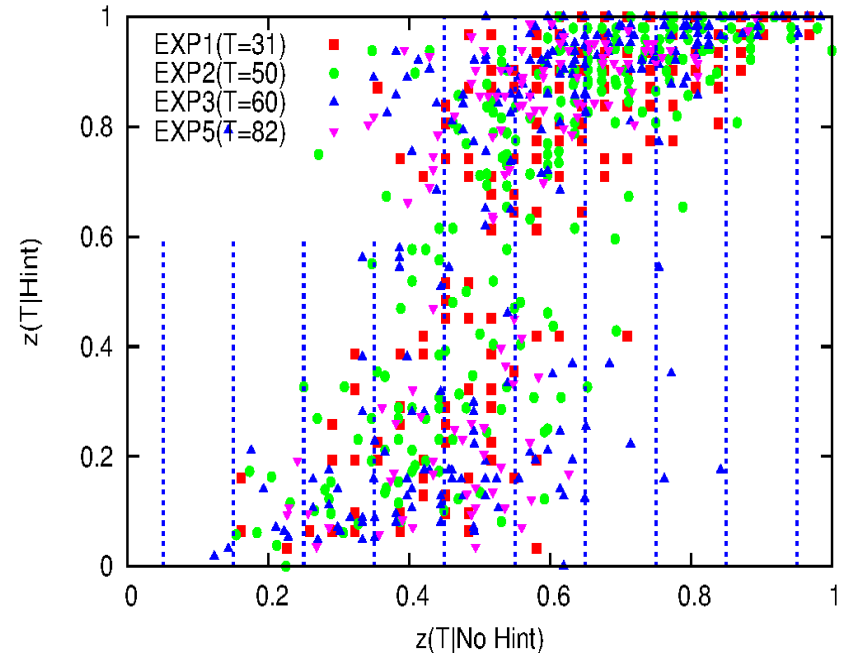
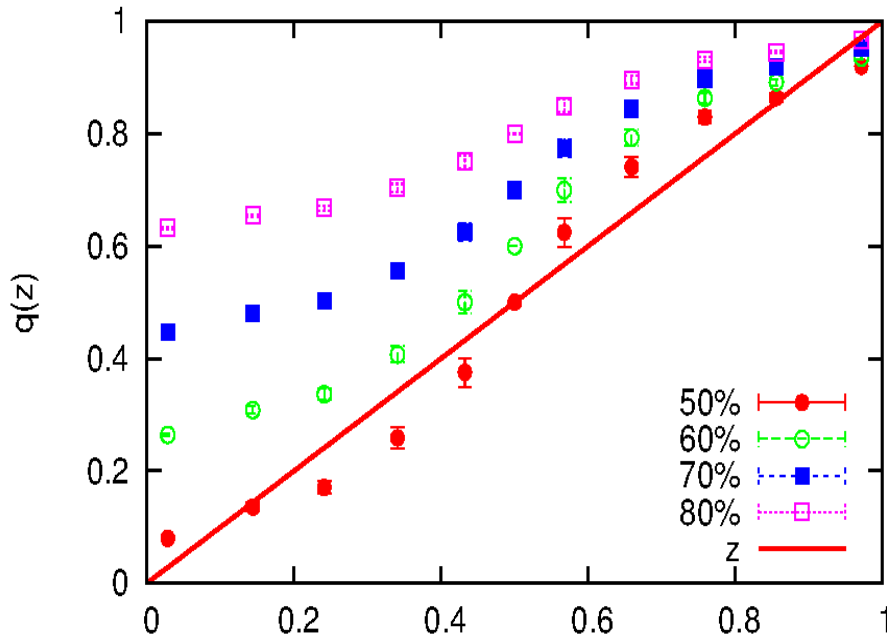
$$q(z) = \Pr(X(t+1) = 1 | z(t) = z)$$



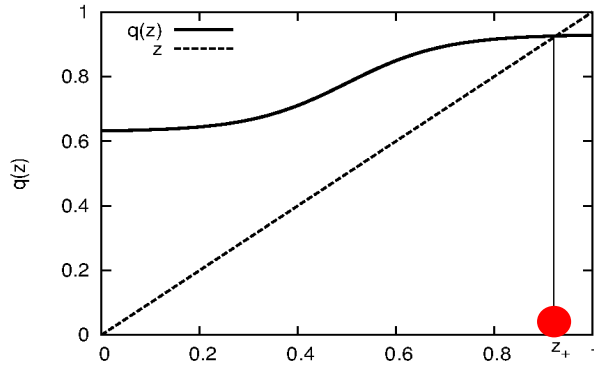
Ratio of correct choices of first t subjects

$$z(t) \equiv \frac{1}{t} \sum_{s=1}^t X(s)$$

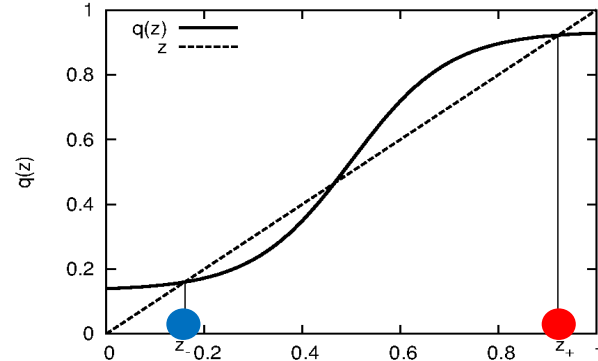
Phase transition of non-linear Pólya urn



Order parameter for non-linear Pólya urn

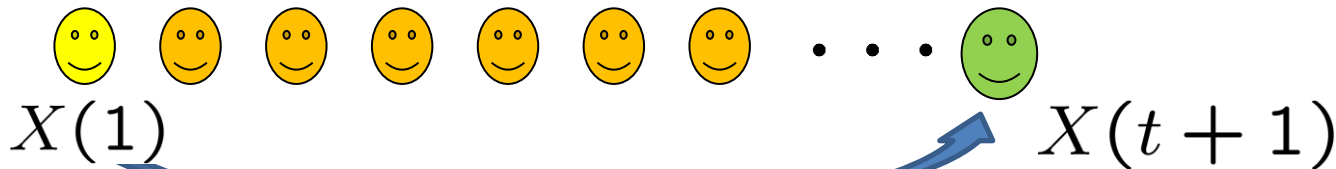


$$\lim_{t \rightarrow \infty} z(t) = z_+$$



$$\lim_{t \rightarrow \infty} z(t) = z_+ \text{ or } z_-$$

C(t): correlation between X(1) and X(t+1)



$$C(t) = \text{Cov}(X(1), X(t+1)) / \text{Var}(X(1))$$

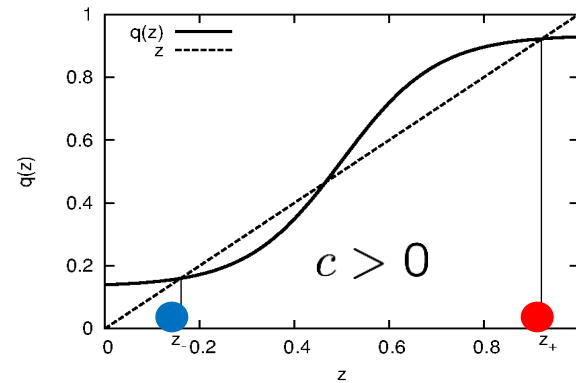
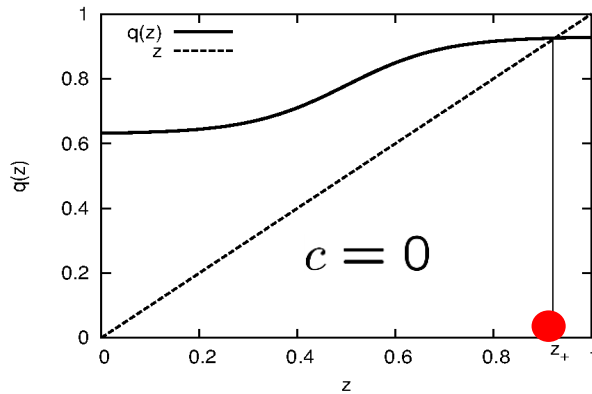
$$C(t) = \text{Pr}(X(t+1) = 1 | X(1) = 1) - \text{Pr}(X(t+1) = 1 | X(1) = 0)$$

Strength of Domino effect

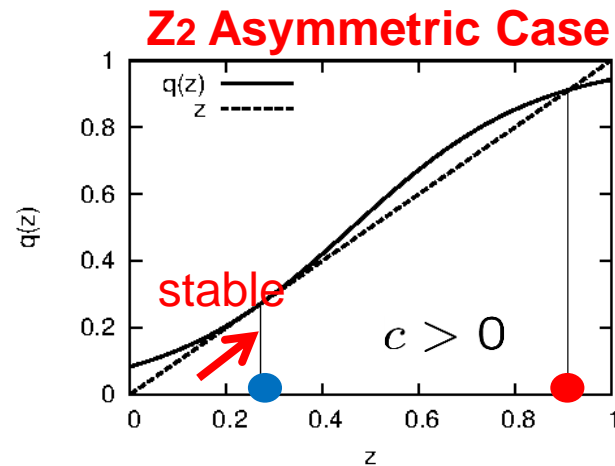
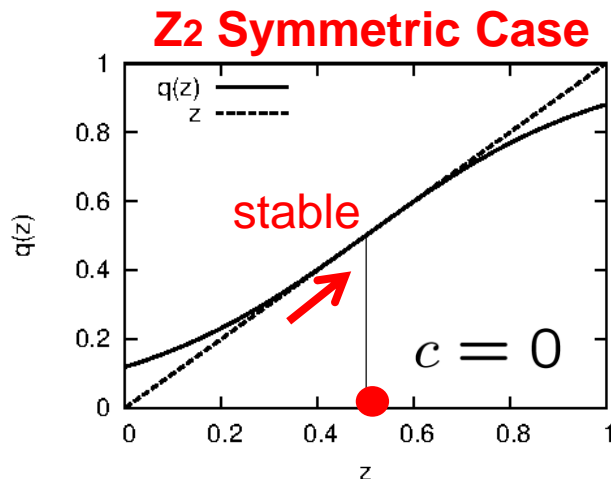
Order parameter for non-linear Pólya urn

$$c = \lim_{t \rightarrow \infty} C(t)$$

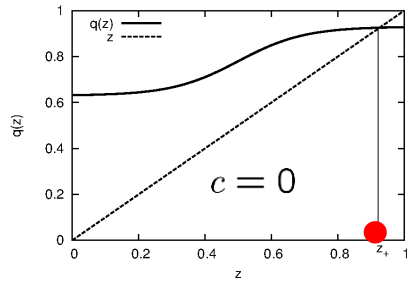
$$C(t) = \Pr(X(t+1) = 1 | X(1) = 1) - \Pr(X(t+1) = 1 | X(1) = 0)$$



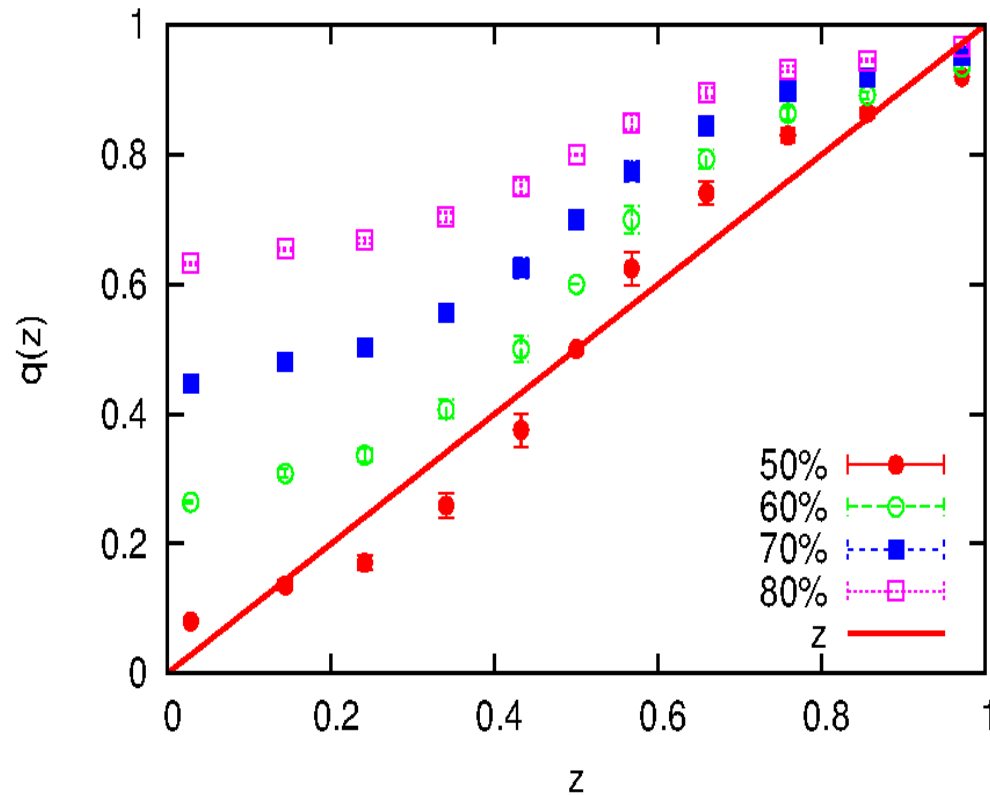
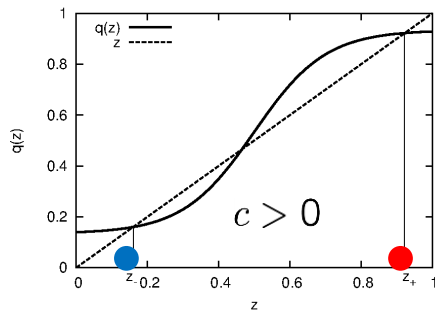
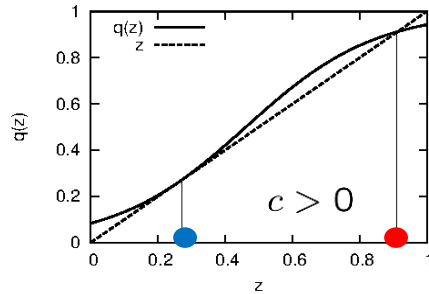
Touchpoint ?



Phase transition in information cascade



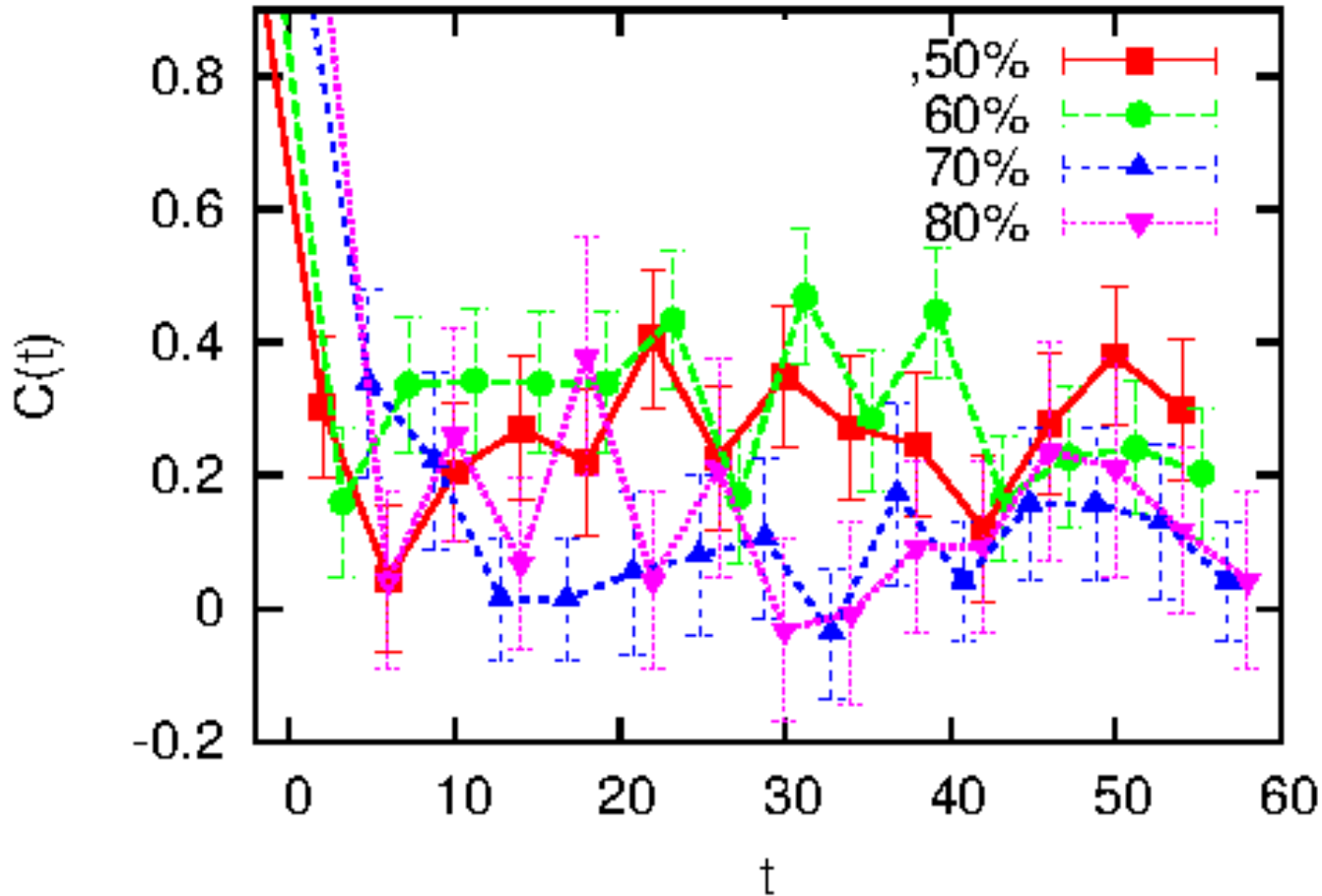
Z2 Asymmetric Case



Discontinuous Transition

Estimation of order parameter c ?

$$C(t) \propto c + c't^{l-1}$$



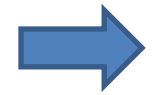
Estimation of c and l

$$C(t) \propto c + c' t^{l-1}$$

Relaxation time

$$\tau(t) = \sum_{s=0}^{t-1} C(s)$$

$$c = 0$$

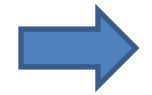


$$\left\{ \begin{array}{l} \tau(t)/t \propto t^{l-1} \rightarrow 0 \\ \xi(t)/t \rightarrow \sqrt{l/(l+2)} < 1/\sqrt{3} \end{array} \right.$$

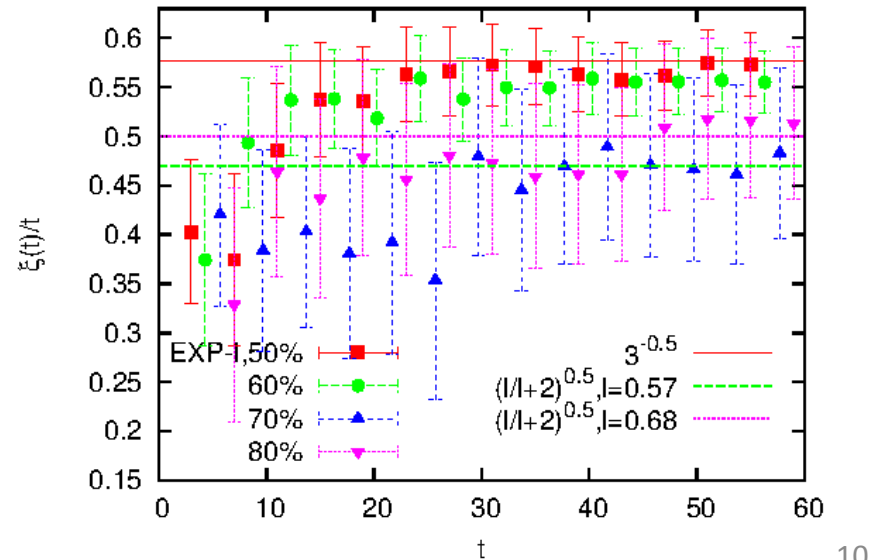
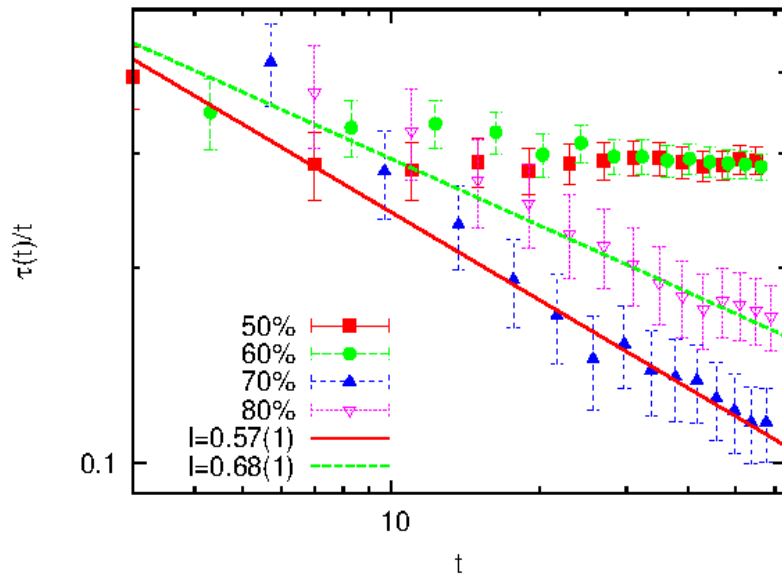
Correlation time

$$\xi(t) = \sqrt{\frac{\sum_{s=0}^{t-1} C(s) s^2}{\sum_{s=0}^{t-1} C(s)}}$$

$$c > 0$$



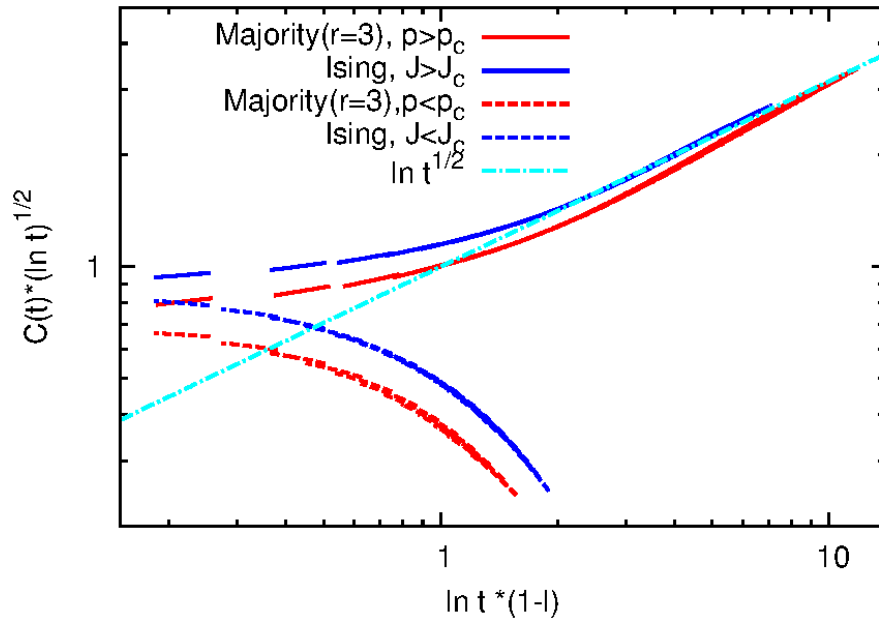
$$\left\{ \begin{array}{l} \tau(t)/t \rightarrow c \\ \xi(t)/t \rightarrow 1/\sqrt{3} \end{array} \right.$$



Universality class of non-linear Pólya urn (Conjecture)

Ising Model $q(z) = \frac{1}{2} [\tanh(J(2z - 1) + h) + 1], h = 0, J_c(0) = 1$

Majority (r=3) $q(z) = (1-p)q + p \cdot (3z^2 - 2z^3), q = 1/2, p_c(q = 1/2) = 2/3$



$$C(t) = (\log t)^{-1/2} g(\log t / \xi_l)$$

$$\xi_l = 1 / (1 - l), l = q'(z_+) = q'(z_-)$$

$$g(x) \propto x^{1/2} \text{ for } J > J_c \text{ or } p > p_c$$

$$\longrightarrow C(t) \propto \xi_l^{-1/2}$$

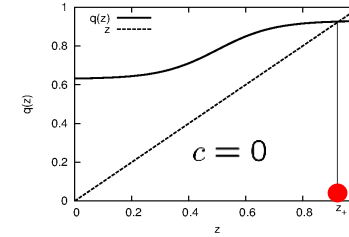
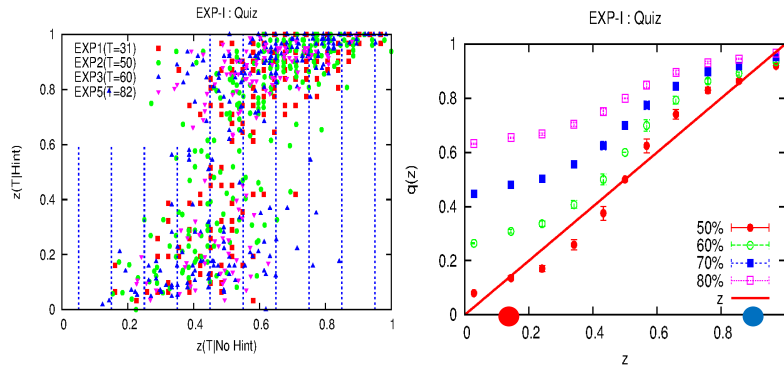
Scaling Relation

$$\left\{ \begin{array}{l} \xi_l \propto \Delta J^{-\nu_{||}}, \Delta p^{-\nu_{||}} \\ C(t) \propto (\log t)^{-\alpha}, \text{ at } J = J_c \text{ or } p = p_c \\ c \propto \Delta J^\beta, \Delta p^\beta \end{array} \right. \longrightarrow \beta = \alpha \cdot \nu_{||}$$

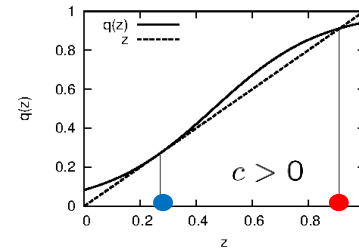
$$\alpha = \frac{1}{2}, \nu_{||} = 1 \longrightarrow \beta = \alpha \cdot \nu_{||} = \frac{1}{2}$$

Summary

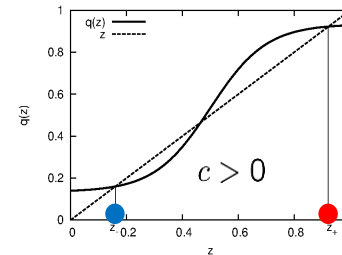
(1) Effective Theory for human in Info.Cascade=Non-linear Pólya Urn



Easy



“Critically” Difficult
Z₂ Asymmetric



Very Difficult

(2) Discontinuous Phase Transition



Order parameter

$$c = \lim_{t \rightarrow \infty} C(t)$$

(3) Universality class (Continuous transition for **Z₂ Symmetric** case)

$$C(t) \simeq (\log t)^{-1/2} g(\log t / \xi_l) \quad \beta = 1/2 \quad c \propto \Delta J^\beta, \Delta p^\beta$$

Phase transition to two-peaks phase in an information cascade voting experiment

S.Mori, M. Hisakado and T. Takahashi, Phys.Rev.E86(2012)026109-026118.

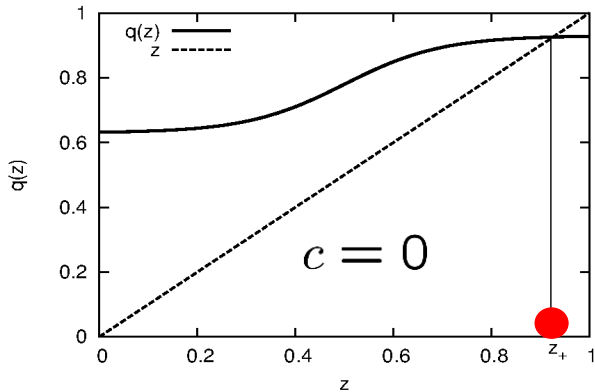
Finite-size scaling analysis of binary stochastic processes and universality classes of information cascade phase transition

S.Mori and M. Hisakado, J. Phys. Soc. Jpn. 84, 054001 (2015)

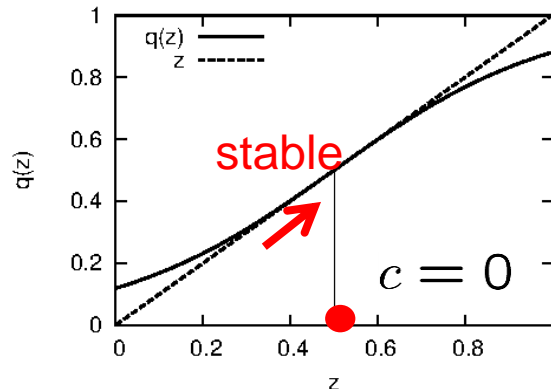
Correlation function for generalized Polya urns: Finite-size scaling analysis

S.Mori and M.Hisakado, arXiv:1501.00764

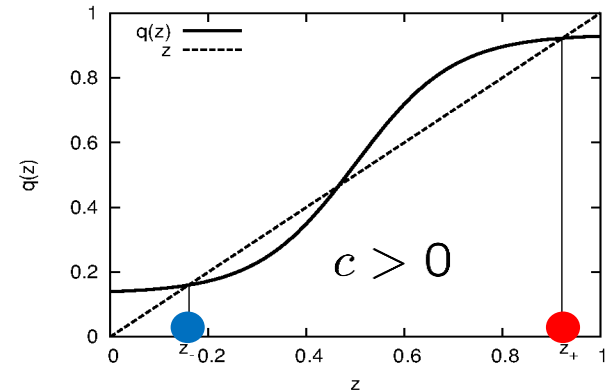
Universality class of non-linear Pólya urn



$$C(t) \simeq c't^{l-1}, l = q'(z_+)$$



$$C(t) \simeq c' \log t^{-1/2}$$



$$C(t) \simeq c + c't^{l-1}, l = q'(z_+)$$

$$t^{l-1} = e^{-(1-l) \log t} = e^{-\log t / \xi_l}, \xi_l = 1/(1-l)$$

$$C(t) \simeq c'e^{-\log t / \xi_l}$$

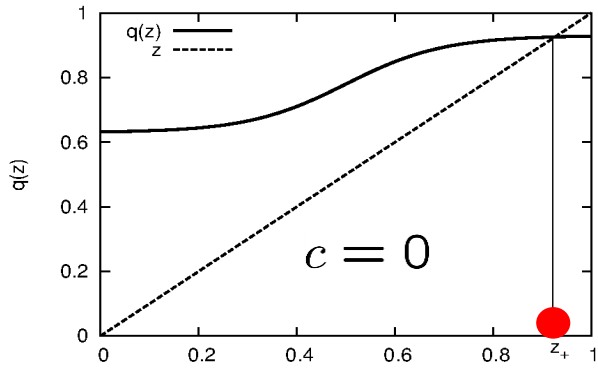
$$C(t) \simeq c' \log t^{-1/2}$$

$$C(t) \simeq c + c'e^{-\log t / \xi_l}$$

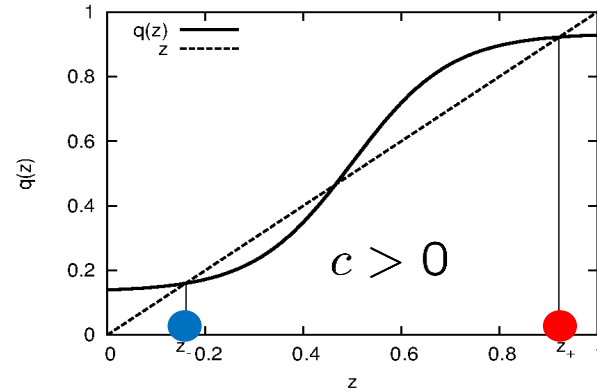
Conjecture for the scaling behavior of $C(t)$

$$C(t) = (\log t)^{-1/2} g(\log t / \xi_l)$$

Asymptotic behavior of C(t)

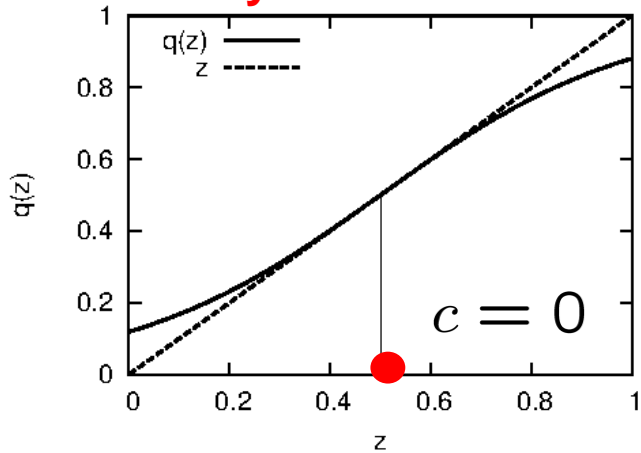


$$C(t) \propto c' t^{z_+ - 1}, \quad l = q'(z_+)$$



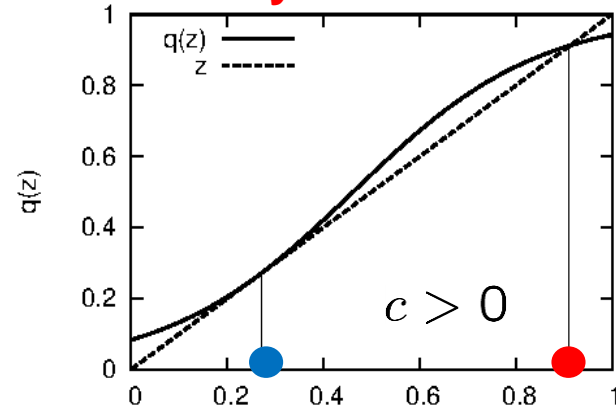
$$C(t) \propto c + c' t^{l-1}, \quad l = \text{Max}(q'(z_+), q'(z_-))$$

Z2 Symmetric Case



$$C(t) \propto c' \log t^{-1/2}$$

Z2 Asymmetric Case



$$C(t) \propto c + c' \log t^{-1}$$