

Phase transition of non-linear Polya urn and Empirical tests in human collectives

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Collaborators

M.Hisakado

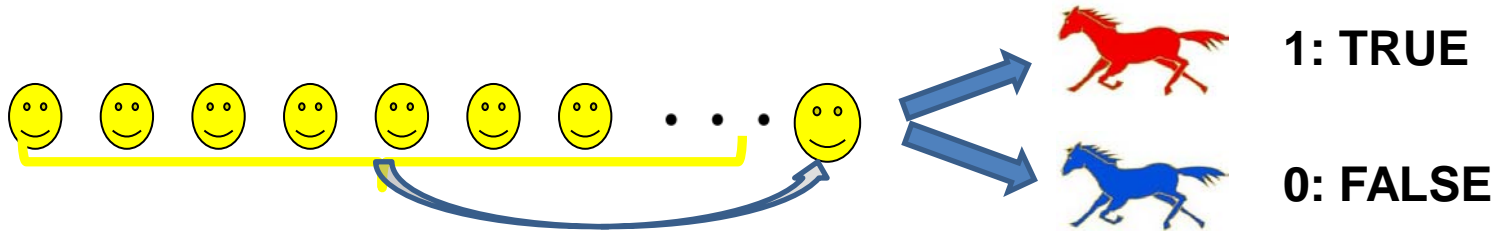
T.Takahash

Information cascade

- (1) X is A or B ? : Discrete Choice.
- (2) Sequential choices, one by one.
- (3) Private signal (secret).
- (4) One watch others' choices .



Choose the majority
 irrespective of one's private signal
 = Information cascade

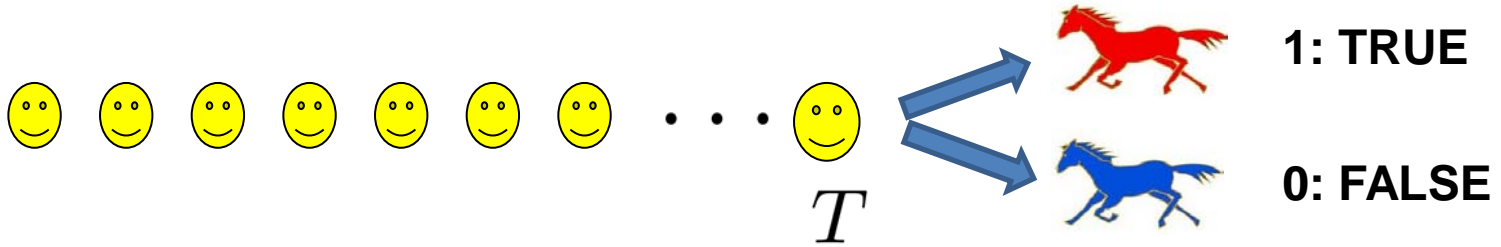


t : order	1	2	3	4	5	6
Private Signal, secret	a	a	b	b	b	b
Choice, public	A	A	A	A	A	A



Information cascade is a tendency to follow the majority. It plays the role of ferromagnetic Interaction between binary decisions of humans.

Experiment 1



Voting Experiment

No. Info.

Q.30: Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky B : Beethoven

Answer

Voting Experiment

All previous subjects' Info.

Up to now 7 subjects have answered.

Their choices are as follows. Please choose.

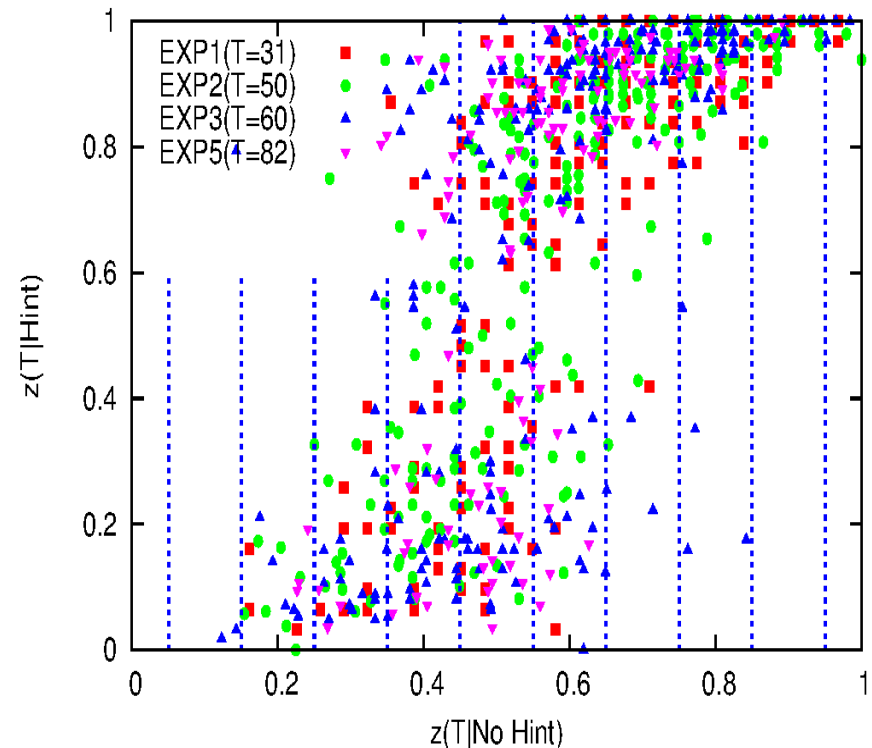
Q.30: Which composer is famous for the Symphonie No.6 Pathetique ?

A : Tchaikovsky B : Beethoven

1人	6人
<input type="radio"/>	<input type="radio"/>

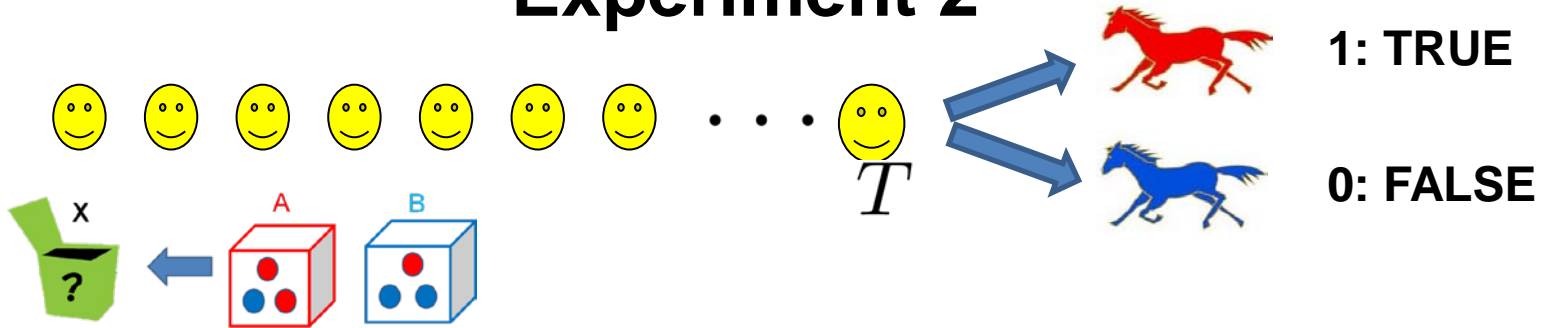
Answer

$Z(T)$: ratio of correct answers



Experimental Setup. Two choice general knowledge question. Length of subject sequence, $T=31 \sim 82$. 120~240 questions. Control parameter is $Difficulty=1-z(T|No Hint)$ of each question.

Experiment 2

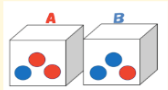


Cascade Experiment

You have answered 5 questions up to now.

Your ball color is **RED**.

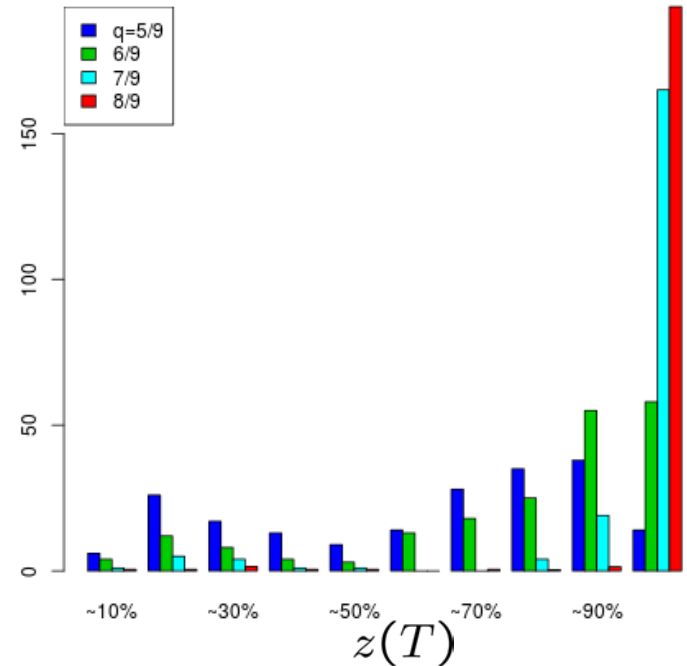
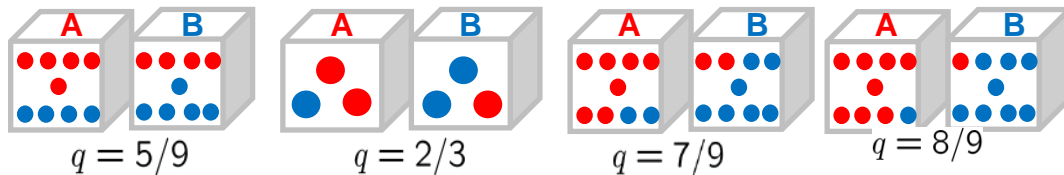
Which type, A or B ?



Please answer your confidence about your choice.

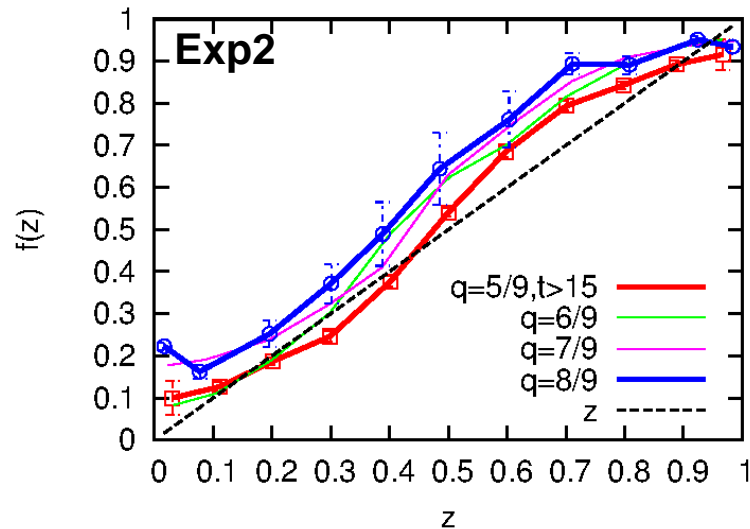
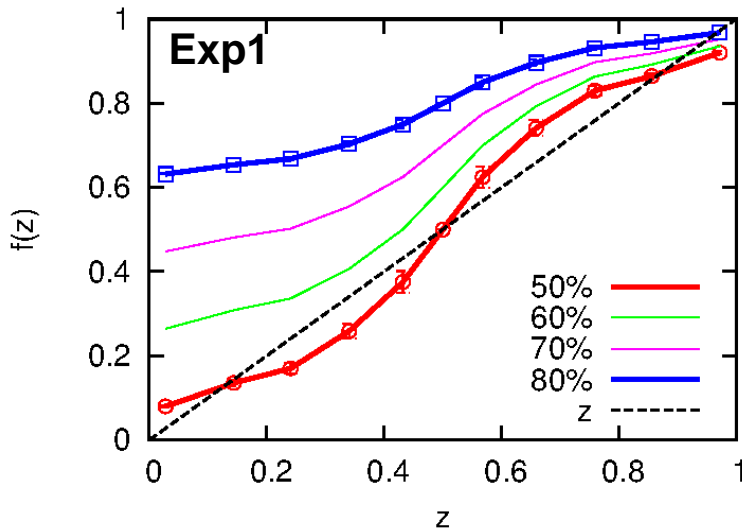
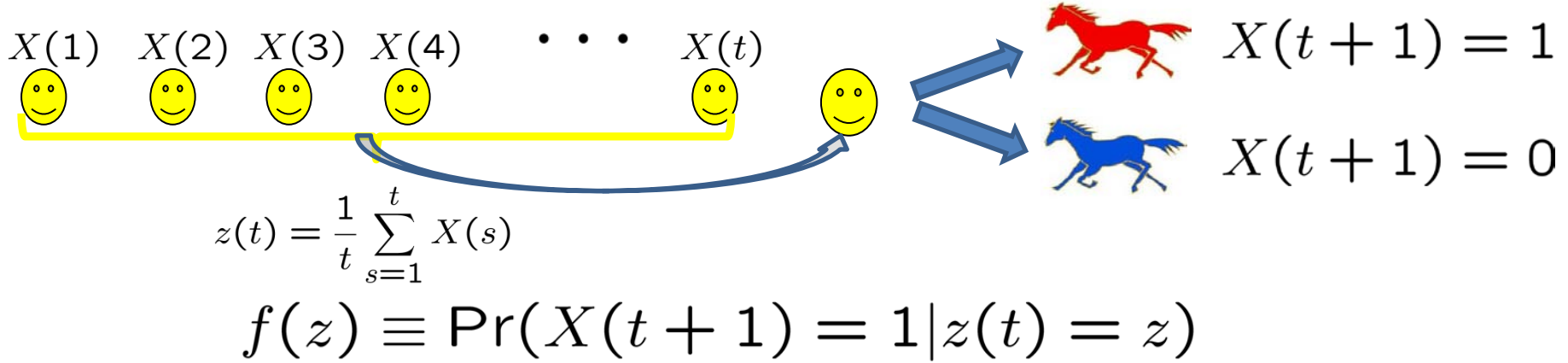
Urn	A						B						Urn
Votes	3						1						Votes
A	100%	90%	80%	70%	60%	50%	50%	60%	70%	80%	90%	100%	B
	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	

Answer



**Experimental Setup. Two choice urn question, Urn X is A or B ?
200 or 400 urn X. Length of subject sequence, T=50~60.
Control parameter is q.**

Information Cascade Interaction



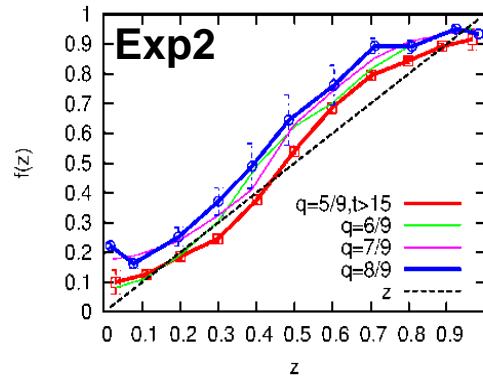
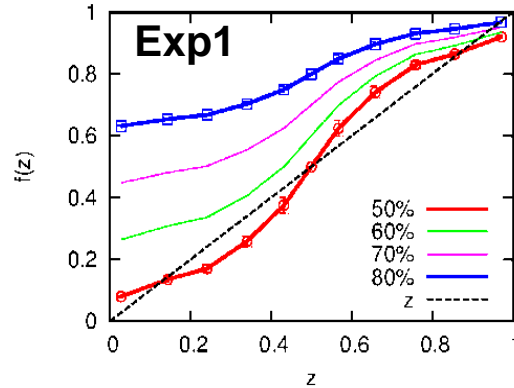
Probabilistic rule of humans' choice influenced by social information. Strong and Positive Correlation. This is a non-linear Pólya Urn process.

Pólya Urn

$X(1), X(2), \dots, X(t) \in \{0, 1\}$ Binary Stochastic Process

$$f(z) : [0, 1] \rightarrow [0, 1] \quad z(t) = \frac{1}{t} \sum_{s=1}^t X(s)$$

$$f(z) = \Pr(X(t+1) = 1 | z(t) = z)$$



G.Pólya(1931)

$$f(z) = z$$

Non-linear Pólya Urn

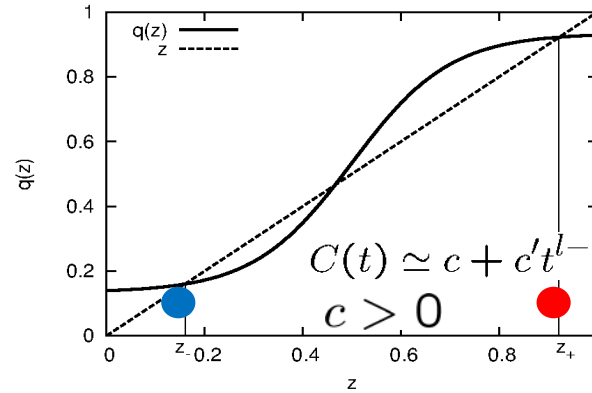
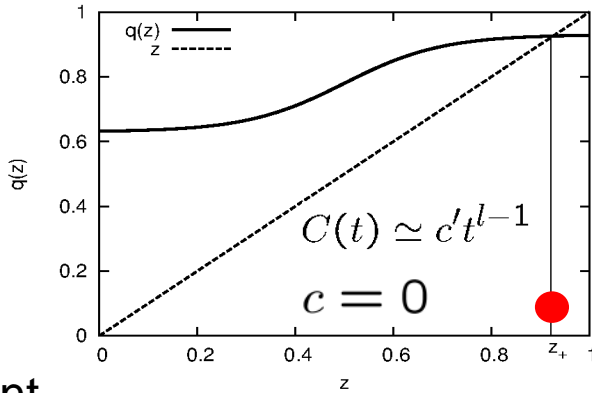
If quiz is **difficult**, there are **two** stable states.

If quiz is **easy**, there is **one** stable state.

The change in the number of stable states is a phase transition.

Order parameter $c = \lim_{t \rightarrow \infty} C(t)$

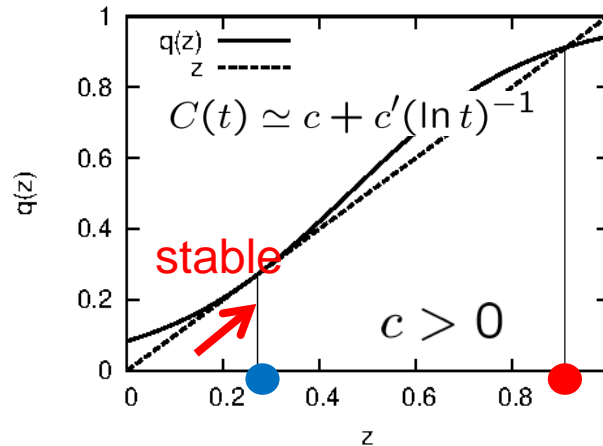
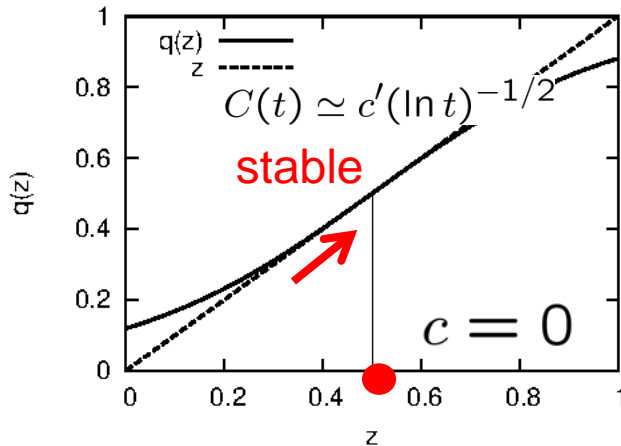
$$C(t) = \Pr(X(t+1) = 1 | X(1) = 1) - \Pr(X(t+1) = 1 | X(1) = 0)$$



Touchpoint

Z2 Symmetric Case

Z2 Asymmetric Case

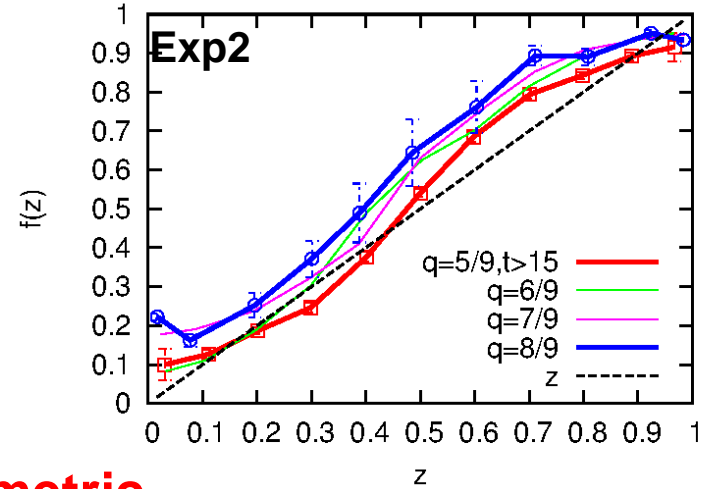
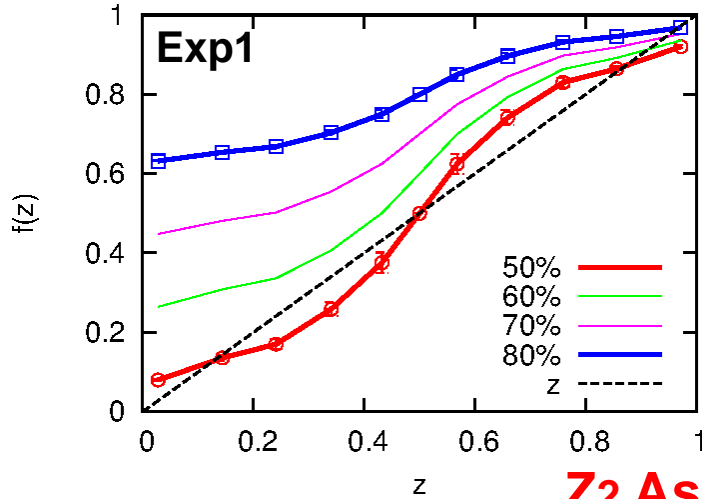


Continuous transition

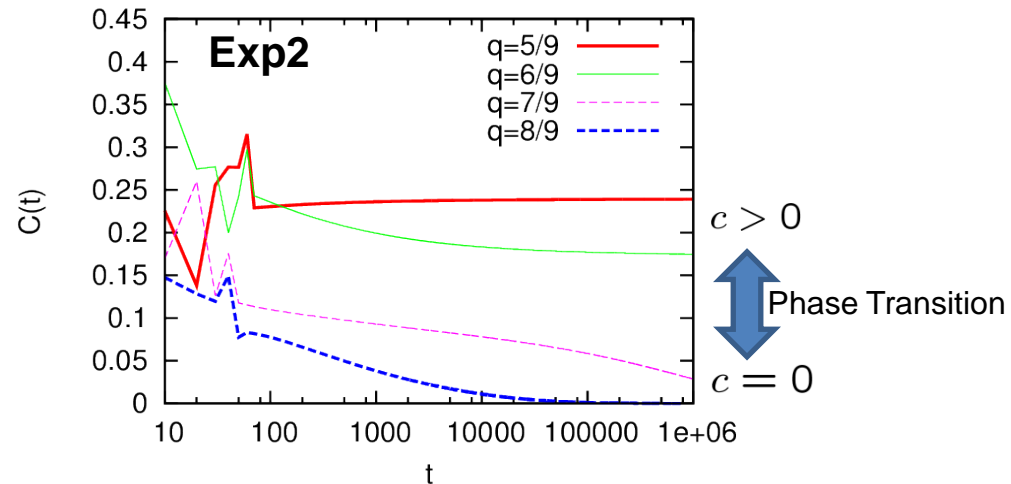
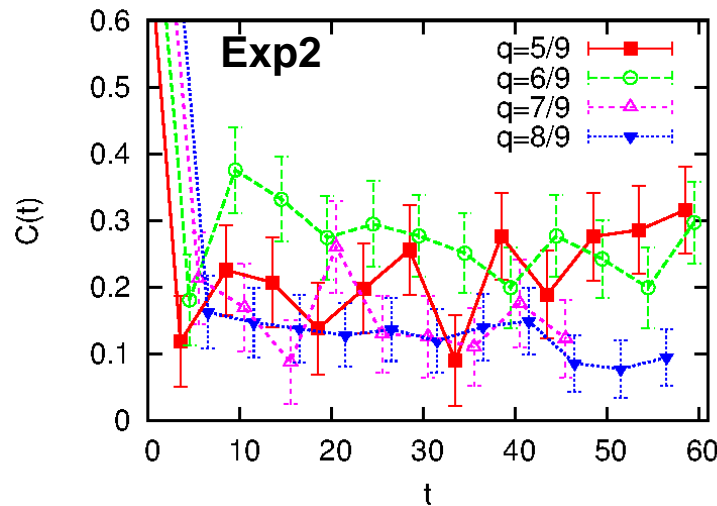
Discontinuous transition

The change in the number of stable states is a Phase transition. Usually, it is discontinuous. Order parameter is the limit value c of $C(t)$. $c > 0$ or $c = 0$ means domino effect continues forever or not ?

Discontinuous transition



Z₂ Asymmetric



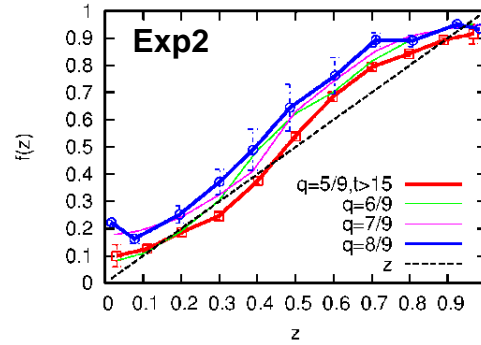
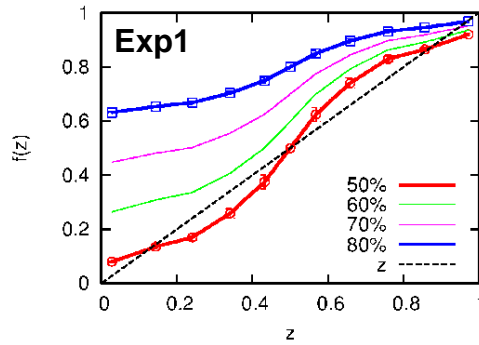
The experimental system is Z₂ asymmetric.

The phase transition is discontinuous.

Order parameter c is estimated by extrapolating $C(t)$ for $t < T$.

Summary

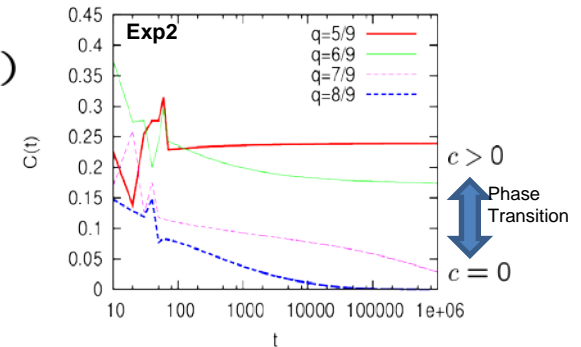
(1) Information cascade → Non-linear Pólya Urn



(2) Discontinuous Phase Transition



Order parameter $c = \lim_{t \rightarrow \infty} C(t)$



(3) Universality class (Z_2 symmetric case)

$$C(t) = (\ln t)^{-\alpha} g(\ln t / \xi_l), \alpha = 1/2$$

$$\left. \begin{aligned} \xi_l &\propto \Delta J^{-\nu_{||}}, \nu_{||} = 1 \\ c &\propto \Delta J^\beta \quad \beta = \alpha \cdot \nu_{||} \end{aligned} \right\} \beta = 1/2$$

References

EXP 1 :S.Mori,M.Hisakado and T.Taakahashi,Phys.Rev.E86(2012)026109 .

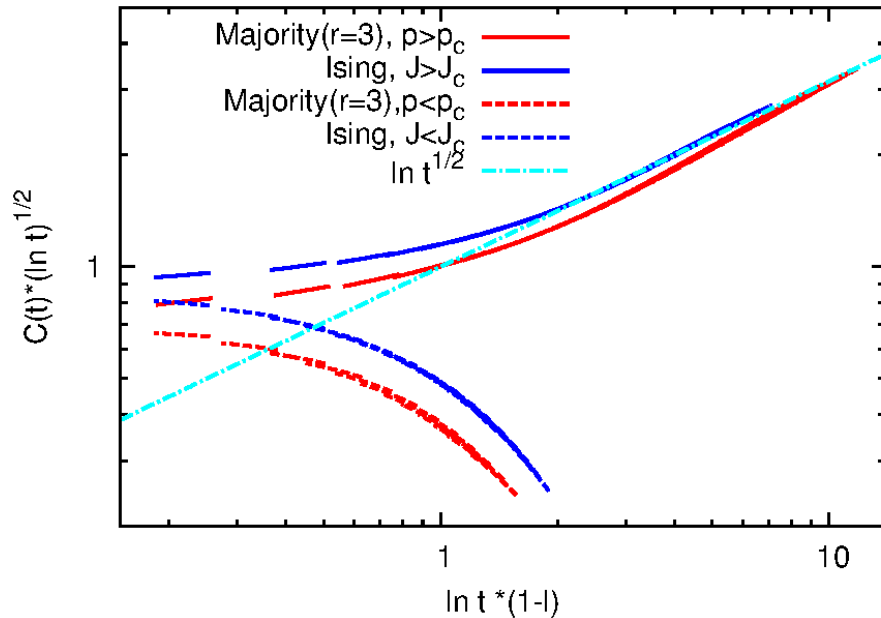
EXP 2: M.Hino, Y.Irie, M.Hisakado, T.Takahasi and S.Mori,J.Phys.Soc.Jpn.86(2016)034002.

Theory: S.Mori and M.Hisakado, Phys.Rev.E92(2015)052112.

Universality class of non-linear Pólya urn

Ising Model $f(z) = \frac{1}{2} [\tanh(J(2z - 1) + h) + 1], h = 0, J_c(0) = 1$

Majority (r=3) $f(z) = (1-p)q + p \cdot (3z^2 - 2z^3), q = 1/2, p_c(q = 1/2) = 2/3$



$$C(t) = (\ln t)^{-1/2} g(\ln t / \xi_l)$$

$$\xi_l = 1/(1-l), l = q'(z_+) = q'(z_-)$$

$$g(x) \propto x^{1/2} \text{ for } J > J_c \text{ or } p > p_c$$

$$\longrightarrow C(t) \propto \xi_l^{-1/2}$$

Scaling Relation

$$\longrightarrow \beta = \alpha \cdot \nu_{||}$$

$$\left\{ \begin{array}{l} \xi_l \propto \Delta J^{-\nu_{||}}, \Delta p^{-\nu_{||}} \\ C(t) \propto (\ln t)^{-\alpha}, \text{ at } J = J_c \text{ or } p = p_c \\ c \propto \Delta J^\beta, \Delta p^\beta \end{array} \right.$$

$$\alpha = \frac{1}{2}, \nu_{||} = 1 \longrightarrow \beta = \alpha \cdot \nu_{||} = \frac{1}{2}$$